Namaste

Jonathan J. Crabtree, Elementary Mathematics Historian, Australia

PART 1. Why India Must Change its Story
Bad Maths History ⇒ Sad Maths Misery

PART 2. Negative & Positive Quantities on a Brahmaguptan Plane for India's Primary Classes

Bad Maths History ⇒ Sad Maths Misery
Why India Must Change its Story

Jonathan J. Crabtree, Elementary Mathematics Historian
www.podometic.in

9th National conference: Technology & Innovations in Math Education
Biennial Conference of the Mathematics Department of the IIT Bombay.
Jointly organized by IISER, Pune & BATU, Lonere.
Pune India, 27th December 2019

I couldn’t make much sense of what I was being taught. So I felt stupid.

I couldn’t make much sense of what I was being taught. So I felt stupid.

“Don’t worry about it, Jonathan.

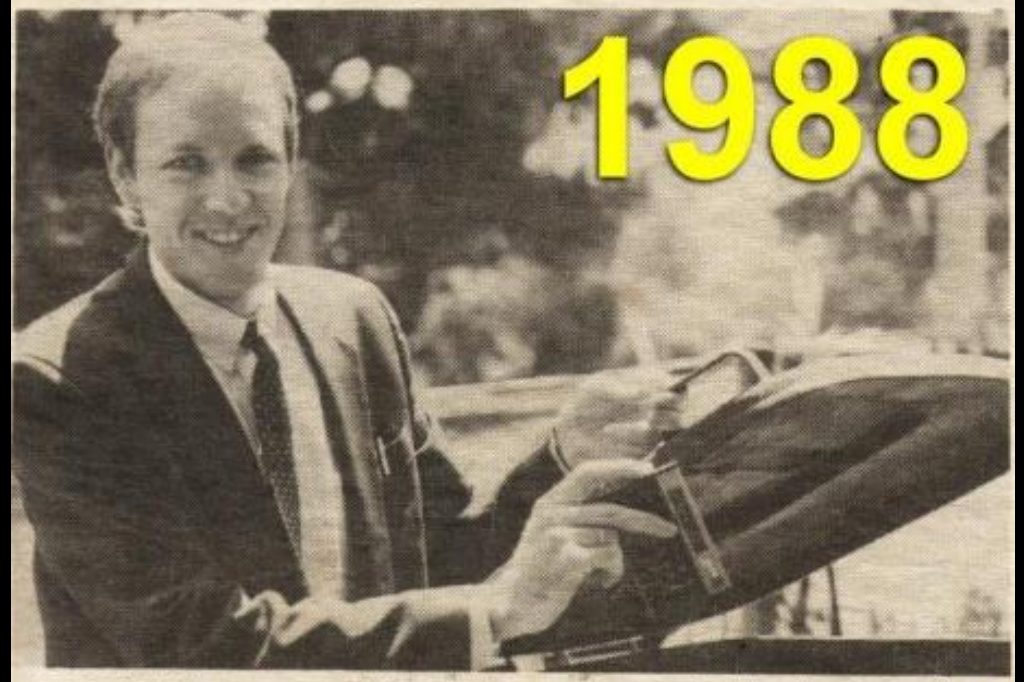
I couldn’t make much sense of what I was being taught. So I felt stupid.

Mathematics is only confusing if you think about it.”

So, in 1983 I made a decision to change primary and middle school mathematics explanations.

So, in 1983 I made a decision to change primary and middle school mathematics explanations.

Maths? It's all in the mind, says Jonathan



JONATHAN throws away his calculator and uses brain powers to solve even the hardest of equations.

IF you were asked what day it was on July 24, 1706, what would you say?

It's all in the mind, he says.

After a four second calculation he came up with the correct day.

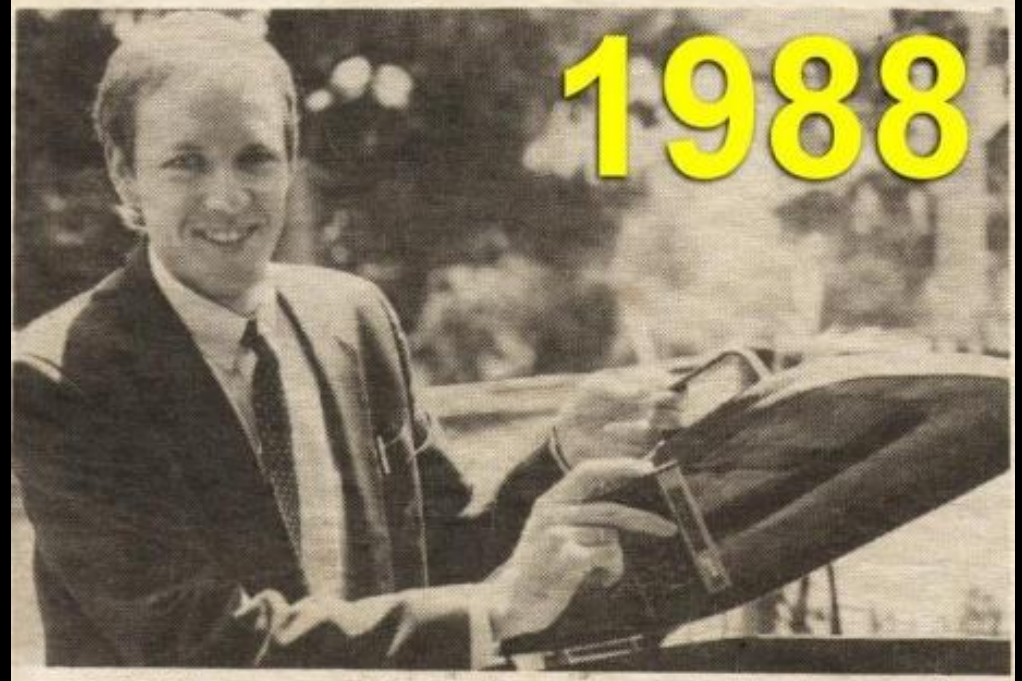
"I hope to change the way the Western world teaches maths," Jonathon said.

Jonathon will be holding two classes at the Park Orchards Community Centre.

The first is *Memory Unlimited*, which

So, in 1983 I made a decision to change primary and middle school mathematics explanations.

Maths? It's all in the mind, says Jonathan



JONATHAN throws away his calculator and uses brain powers to solve even the hardest of equations.

IF you were asked what day it was on July 24, 1706, what would you say?

It's all in the mind, he says.

After a four second calculation he came up with the correct day.

"I hope to change the way the Western world teaches maths," Jonathon said.

Jonathon will be holding two classes at the Park Orchards Community Centre.

The first is *Memory Unlimited*, which

"I hope to change the way the Western world teaches maths," Jonathon said.

At age seven in Class 2, I pulled on a loose thread.

The multiplication explanation my teacher Miss Collins gave had been wrong for 398 years.

At age seven in Class 2, I pulled on a loose thread.

The multiplication explanation my teacher Miss Collins gave had been wrong for 398 years.

So, that classroom confusion in 1968
led to a peer-reviewed paper.

The Lost Logic Of Elementary Mathematics And The Haberdasher Who Kidnapped Kaizen

Jonathan Crabtree

Download the paper @ www.bit.ly/LostLogicOfMath

www.jonathancrabtree.com | Mathematics Historian

Euclid's multiplication definition from Elements, *(c. 300 BCE), continues to shape mathematics education today. Yet, upon translation into English in 1570 a 'bug' was created that slowly evolved into a 'virus'. Input two numbers into Euclid's step-by-step definition and it outputs an error. Our multiplication definition, thought to be Euclid's, is in fact that of London haberdasher, Henry Billingsley who in effect kidnapped kaizen, the process of continuous improvement. With our centuries-old multiplication definition revealed to be false, further curricular and pedagogical research will be required. In accordance with the Scientific Method, the Elements of western mathematics education must now be rebuilt upon firmer foundations.*

At age seven in Class 2, I pulled on a loose thread.

The multiplication explanation my teacher Miss Collins gave had been wrong for 398 years.

So, that classroom confusion in 1968
led to a peer-reviewed paper.

The more I pulled on loose threads, the more mathematics education unraveled.

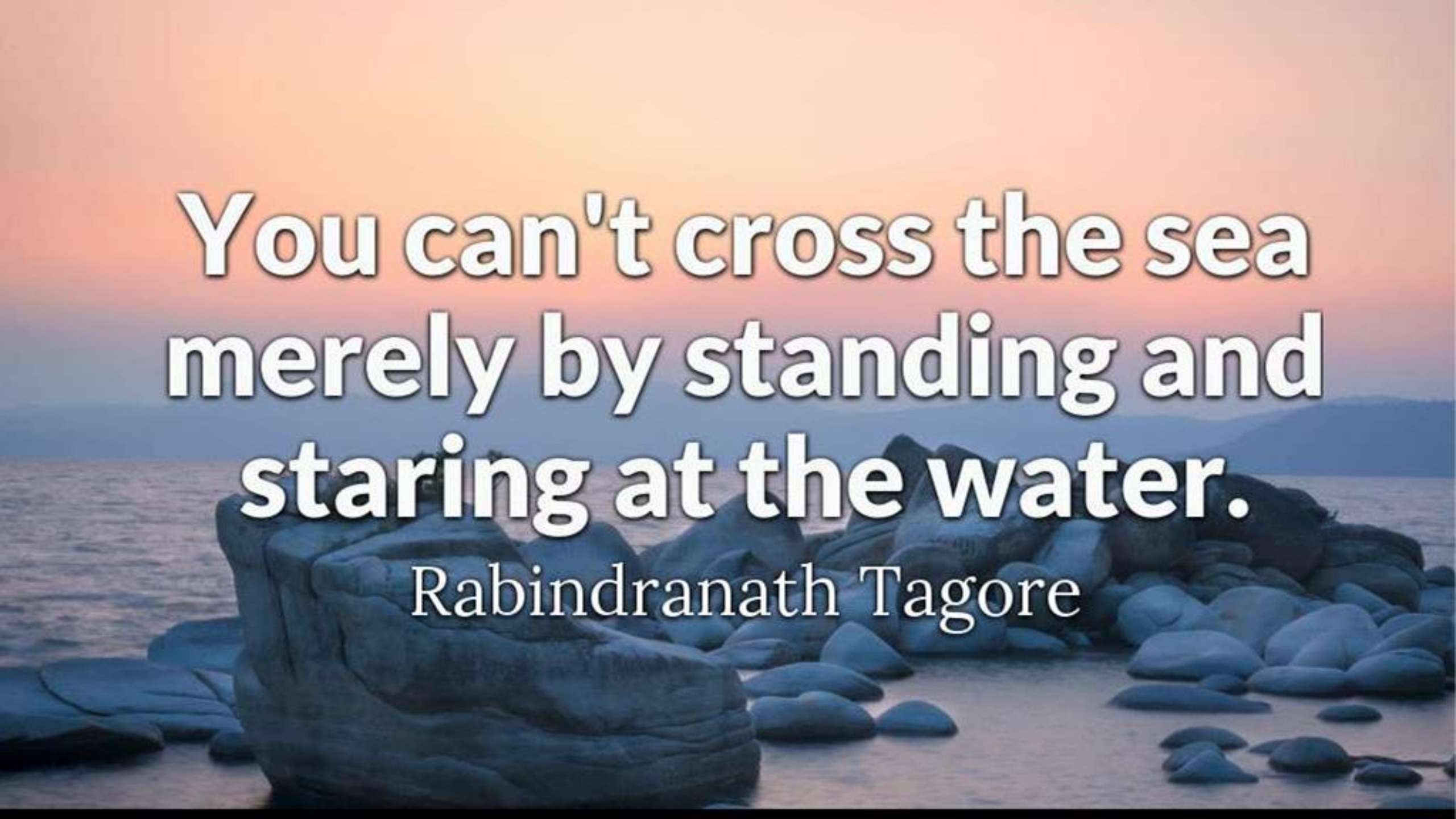
INDIA WE HAVE A PROBLEM

Many Dislike or Fear Your Maths

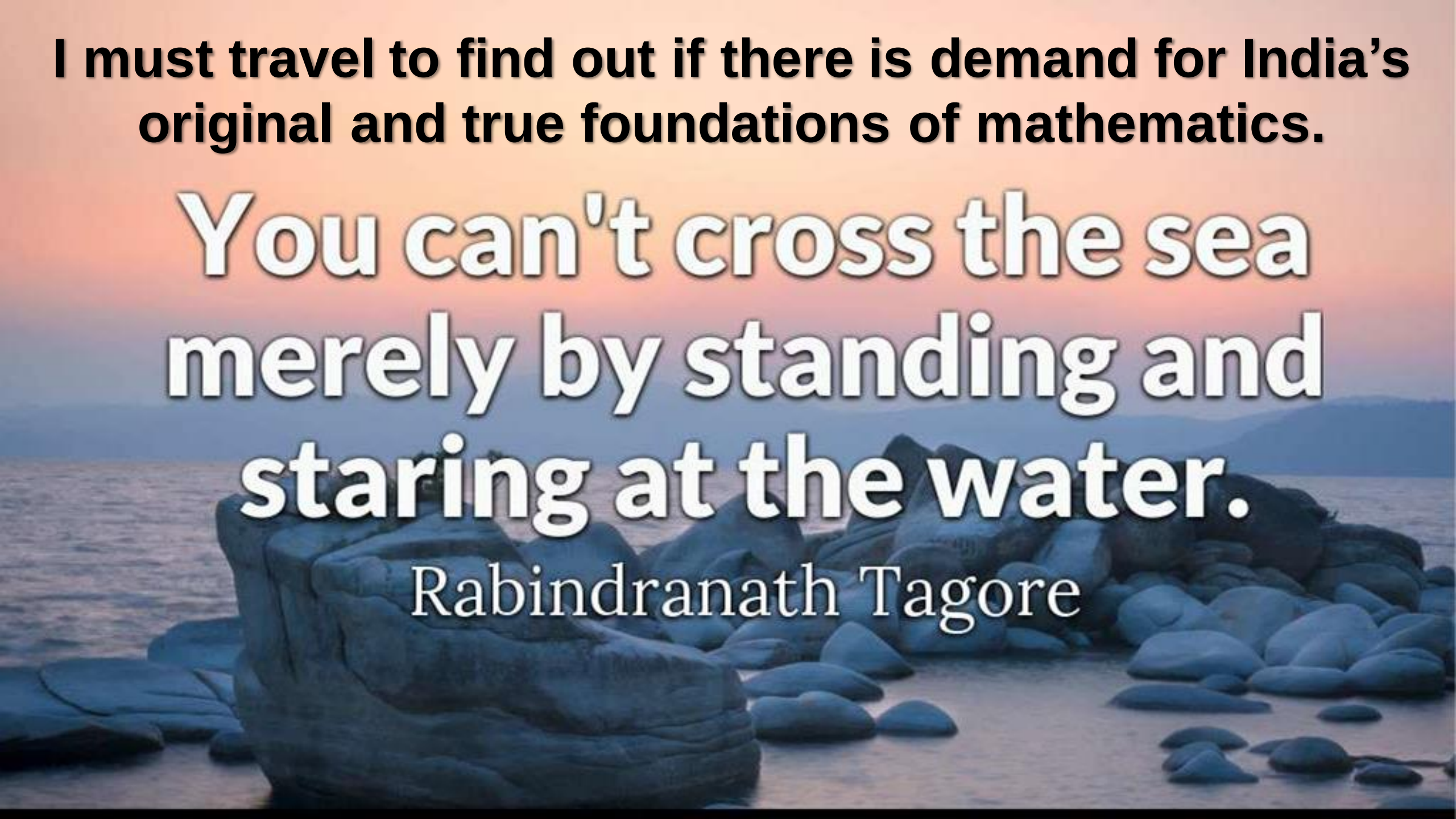
INDIA WE HAVE A PROBLEM

Many Dislike or Fear Your Maths

What happened?


You can't cross the sea merely by standing and staring at the water.
Rabindranath Tagore

I must travel to find out if there is demand for India's original and true foundations of mathematics.



WELCOME

প্রভাত খবর

রিলাংচ অফ ইন্ডিয়ন মৈথমেটিক্স পর সংগোষ্ঠী

কোলকাতা. এসোসিয়েশন অফ ইন্টারনেশনাল মৈথেমেটিক্স এডুকেশন এন্ড রিসর্চ এর অধ্যক্ষ ডাঁ এসআর সৈবনম কী অধ্যক্ষতা মেঁ শনিবার কো 'रिलांच ऑफ इंडियन मैथेमेटिक्स' पर एक संगोष्ठी का आयोजन किया गया. जिसमें गणित इतिहासकार जोनाथेन जे. क्रैबी को आमंत्रित किया गया. उन्होंने शून्य दिया और आज भारत ही गणित के मामले में विश्व के 72वें स्थान पर आता है. दुनिया के सबसे बड़े लोकतंत्र के रूप में भारत अत्यधिक सफलता का हकदार है और यही कारण है कि आज भी भारतीय को गणित की खोज करनेवालों में पहला व्यक्ति होना चाहिए, लेकिन अभी वहां के अधिकतर बच्चे

বেঙ্গল ফ

শ্রীনিবাস রামানুজন
জন্ম ২২ ডিসেম্বর ১৮৮৭
মৃত্যু ২৬ এপ্রিল ১৯২০

রামানুজনের জন্মদিনে সহজ গণিত পদ্ধতি উপহার

রাজ্যের প্রতিটি স্কুলে বিনামূল্যে সহজ গণিত শিক্ষার বই দিতে চান অস্ট্রেলিয়ার গণিতজ্ঞ

আজকাল

রবিবার

ভীতি কাটাতে শহরে গণিতজ্ঞ

আজকালের প্রতিবেদন

জোনাথন জে ক্রাবট্রি

FOTO FACT NEWS

Volume-I, Issue- 49 (Weekly) • Kolkata Edition • 12-November-18November 2018 • RNI No. WBBEN/2017/75060 • Rs. 2/-

Jonathan J. Crabtree Elementary Mathematics Historian from Melbourne addressing in connection to organise a seminar on "Relaunch of Indian Mathematics" at Rotary Sadan, in association with Indian Red Cross Society and AFX Animation. (Pic. Md. Iqbal Khan)

গাণিতিক ফর্মুলা চালু হয়েছে বিশ্বের ষাটটি দেশে। কলকাতায় অনুষ্ঠিত হচ্ছে দেশের সবচেয়ে বড় গণিতমেলা। জোনাথন শেখাবেন সহজ গণনা পদ্ধতি

১ থেকে ৫ গুনতে জানলেই জটিল অঙ্ক হবে সহজ, কাটবে ভীতি

সৌগত সরকার

সেই খুড়ো আজ কল করেছেন আপন বুদ্ধি বলে
পাঁচ ঘণ্টার রাস্তা যাবেন দেড় ঘণ্টায় চলে
দেখে এলাম কলটি অতি সহজ এবং সোজা
ঘণ্টা পাঁচেক মাঁটলে পরে আপনি যাবে বোঝা।।

সুকুমার রায়ের 'খুড়োর কল' ছড়ায় চণ্ডীদাসের খুড়ো এমনই এক আজব কল লাগিয়েছিলেন। যদিও সেই কল ছিল খুড়োর মস্তিষ্কের থেকেও জটিল। কিন্তু সত্যি যদি এমন কোনও কল থাকত, জটিলতম সমস্যা এক তুড়িতেই গলে জল। বৈজ্ঞানিক গণনায় দেখা গিয়েছে— পৃথিবীর নব্বই শতাংশ পড়ুয়াই অঙ্ককে ভয় পায়, সে ভয়ের কারণ শুধুমাত্র অঙ্ক কষার জটিলতা। প্রায় সাড়ে তিন হাজার বছর ধরে মানব জীবনে গণিতকে কাজে লাগানোর জন্য গণিতিরা মাথা ঘামিয়ে আসছেন। সভ্যতা যত এগিয়েছে গণিতের প্রয়োজনও তত বেড়েছে। সেই সঙ্গে

"Brahmagupta's 18 laws of mathematics are completely missing from India's present mathematics curriculum."

Jonathan J. Crabtree

Q Why and when did you feel that there were some mistakes in basic mathematical concepts?

A. Fifty years ago, in 1968, my Class 2 teacher gave me the

তারপর রোটারি সদনে শতাধিক অঙ্কপ্রিয় ব্যক্তি ব্যাপ্তা করবেন জোনাথন। এএফএক্স নিজের ব্যক্তিগত উদ্যোগে এই মহান এবং জনা। জোনাথন জানিয়েছেন, তাঁর গবি ব্রাটক ভারতীয় গণনাশাস্ত্র এবং গণিতশাস্ত্র। প্রাচীন ভারতীয় পুথি। পাঁচ বছর এই পুথি গুলো বিশ্ববিদ্যালয়ের লাইব্রেরি থেকে। দেখা অঙ্কশাস্ত্রের এই লুপ্তির পুথিটির শূন্যের ব্যবহার, ধনাত্মক ও ঋণাত্মক গণনা পুথির মূল বিষয়। প্রাচীন ভারতীয় গণিতজ্ঞ ণ্ড-এর সেই গণিতশাস্ত্রেই ভারিত হয়েছে। নতুন গণনা পদ্ধতি ইতিমধ্যেই পৃথিবীর লক ছাত্রে তা চালু রয়েছে। সম্প্রতি তাঁর এই ক্ষিক সম্মান (ফিল্ডস মেডেল) যা নোবেলের পক্ষা রয়েছে। কিন্তু জোনাথন এবং এএফএক্স থেকে বড় পুরস্কার সেটাই হবে যখন সব হজ গণনা পদ্ধতি অঙ্কটে সহজ করে দেবে।

Arithmetic?

Basic Operations (+, –, ×, ÷) on

Arithmetic?

Basic Operations (+, –, ×, ÷) on

… –3 –2 –1 0 +1 +2 +3 …

Obviously, India's ancient integer logic got to us today.

Obviously, India's ancient integer logic got to us today.

But how?

What FACTS do we know?

What FACTS do we know?

India’s definition of zero as a number and Integer arithmetic was embraced by the Arabic world HAN

What FACTS do we know?

India’s definition of zero as a number and Integer arithmetic was embraced by the Arabic world HAN

Al-Khwārizmī wrote a book on Hindu Integer arithmetic which featured Brahmagupta’s ancient laws of sign for negatives and positives

What FACTS do we know?

India's definition of zero as a number and Integer arithmetic was embraced by the Arabic world HAN

Al-Khwārizmī wrote a book on Hindu Integer arithmetic which featured Brahmagupta's ancient laws of sign for negatives and positives

Based on what he learned from the Indians, al-Khwārizmī then wrote a book on algebra

Al-Khwārizmī's **algebra** text c. 820 CE

- لكتاب المختصر في حساب الجبر والمقابلة
- Al-Kitāb al-mukhtasar fī hisāb **al-jabr** wa'l-muqābala
- The Compendious Book on Calculation by Completion and Balancing.

What FACTS do we know?

Arabic writers understood **negative** terms. E.g.

In mathematical language, the verb [jabr] means…
… to transpose **negative** quantities to the opposite side by changing their signs. The **negative** quantity thus removed…

Rosen 1831: p. 178

What FACTS do we know?

Arabic writers understood **negative** terms. E.g.

The usual meaning of jabr in mathematical treatises is: adding equal terms to both sides of an equation in order to eliminate **negative** terms.

Van der Waerden 1985: p. 4

What FACTS do we know?

Arabic writers understood **negative** terms. E.g.

Al-jabr means “restoration” or “completion”, that is, removing **negative** terms, by transposing them to the other side of the equation to make them positive

Devlin 2012: p. 53

What FACTS do we know?

From the Arabic world, India's mathematical foundations made their way to North Africa where Leonardo Pisano (AKA Fibonacci) mastered them

What FACTS do we know?

From the Arabic world, India's mathematical foundations made their way to North Africa where Leonardo Pisano (AKA Fibonacci) mastered them

Leonardo Pisano then documented India's mathematical foundations involving Brahmagupta's definition of zero as a number

What FACTS do we know?

From the Arabic world, India's mathematical foundations made their way to North Africa where Leonardo Pisano (AKA Fibonacci) mastered them

Leonardo Pisano then documented India's mathematical foundations involving Brahmagupta's definition of zero as a number

Thus, Europe came to understand Indian arithmetic

The Transmission of Bharatan (Indian) Integer Arithmetic

Shadings for illustrative purposes only | © 2020 J. J. Crabtree | All Rights Reserved | www.jonathancrabtree.com

Bharat / India
7th century

Teach Brahmagupta's Better Bharatan Maths! Sign the Petition at www.j.mp/BharatanMaths

The Transmission of Bharatan (Indian) Integer Arithmetic

Shadings for illustrative purposes only | © 2020 J. J. Crabtree | All Rights Reserved | www.jonathancrabtree.com

Bharat / India
7th century

Arabic World
9th century

Teach Brahmagupta's Better Bharatan Maths! Sign the Petition at www.j.mp/BharatanMaths

The Transmission of Bharatan (Indian) Integer Arithmetic

Shadings for illustrative purposes only | © 2020 J. J. Crabtree | All Rights Reserved | www.jonathancrabtree.com

Bharat / India
7th century

Arabic World
9th century

Europe
13th century

Teach Brahmagupta's Better Bharatan Maths! Sign the Petition at www.j.mp/BharatanMaths

What FACTS do we teach?

Zero is defined as $n - n$

What FACTS do we teach?

Zero is defined as $n - n$

Negative numbers are less than zero

What FACTS do we teach?

Zero is defined as $n - n$

Negative numbers are less than zero

Negative seven is less than negative four $-7 < -4$

What FACTS do we know?

Zero is defined as $n - n$

Negative numbers are less than zero

Negative seven is less than negative four $-7 < -4$

Every basic arithmetical operation (+, –, ×, ÷) on the Integers is understood and has been for centuries

What <u>FACTS</u> do we teach?

Euclid in his book *Elements* defined multiplication as repeated addition

What FACTS do we teach?

Euclid in his book *Elements* defined multiplication as repeated addition

ab is thus defined as a added to itself b times

What FACTS do we teach?

Euclid in his book *Elements* defined multiplication as repeated addition

ab is thus defined as a added to itself b times

a^b is thus defined as a into itself b times

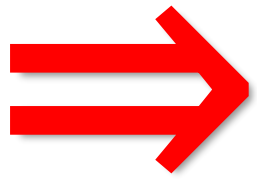
What we know about arithmetic isn't true!

What we know about arithmetic isn't true!

Every previous 'FACT' is FALSE!

Bad Maths History

Bad Maths History



Sad Maths Misery

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

Printed from
THE TIMES OF INDIA

Indian students rank 2nd last in global test

TNN | Jan 15, 2012, 02.24 AM IST

School students celebrate after checking their CBSE results. A global survey has found that the average 15-yea... Read More

MUMBAI: Across the world, India is seen as an education powerhouse — based largely on the reputation of a few islands of academic excellence such as the IITs. But scratch the glossy surface of our education system and the picture turns seriously bleak.

Fifteen-year-old Indians who were put, for the first time, on a global stage stood second to last, only beating Kyrgyzstan when tested on their reading, math and science abilities.

India ranked second last among the 73 countries that participated in the Programme for International Student Assessment (PISA), conducted annually to evaluate education systems worldwide by the OECD (Organisation for Economic Co-operation and Development) Secretariat. The survey is based on two-hour tests that half a million students are put through.

Tamil Nadu and Himachal, showpieces of India's education and development, fared miserably at the Programme for International Student Asssment, conducted by the Organisation for Economic Co-operation and Development Secretariat.

In math, considered India's strong point, they finished second and third to last, beating only Kyrgyzstan

Bad Maths History

Sad Maths Misery

Everyone is entitled to their own opinion, yet not to their own facts.

Extraordinary Claims

- India's definition of **ZERO** never made it to either the ancient Arabic world or Europe.

Extraordinary Claims

- India's definition of **ZERO** never made it to either the ancient Arabic world or Europe.

- In the Arabic world, India's **ZERO** only came to exist as a placeholder, not as the power tool to solve simple problems like $^{+}3$ minus $^{+}4$, or $^{-}2$ minus $^{-}4$, or $^{-}4$ minus $^{+}2$

© 2019 J. J. Crabtree | www.jonathancrabtree.com

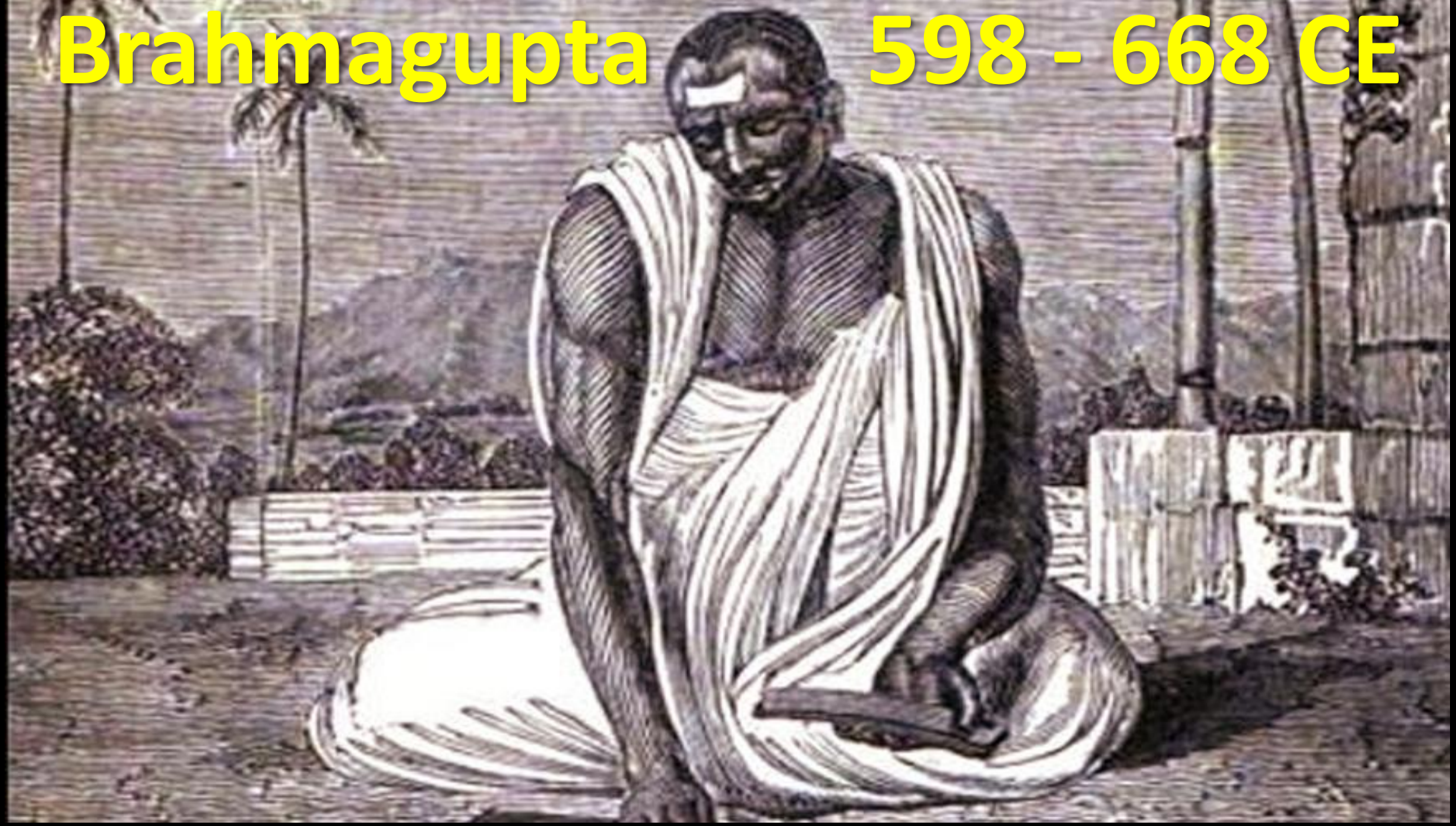
Extraordinary Claims

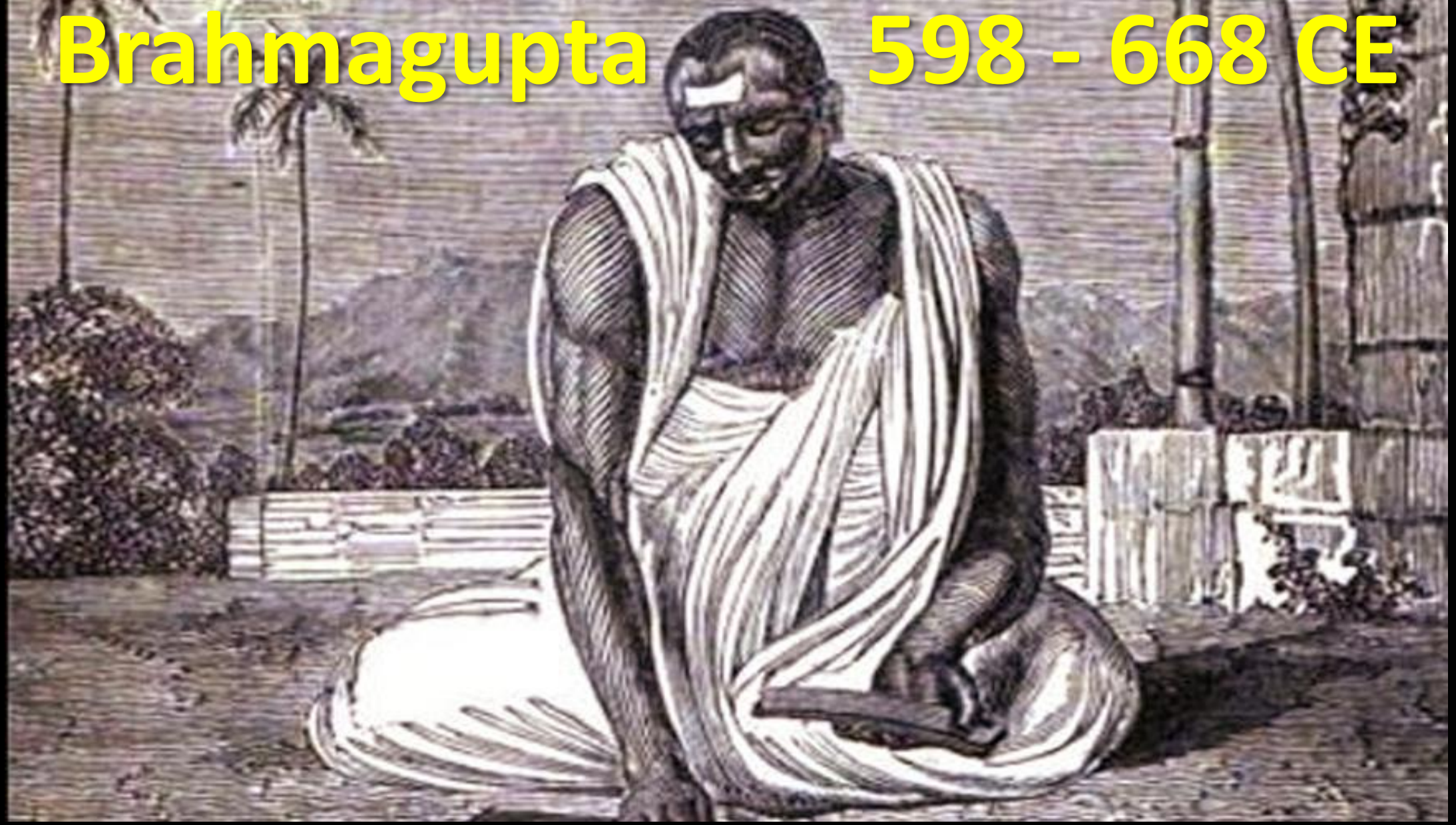
- India's definition of **ZERO** never made it to either the ancient Arabic world or Europe.

- In the Arabic world, India's **ZERO** only came to exist as a placeholder, not as the power tool to solve simple problems like $^{+}3$ minus $^{+}4$, or $^{-}2$ minus $^{-}4$, or $^{-}4$ minus $^{+}2$

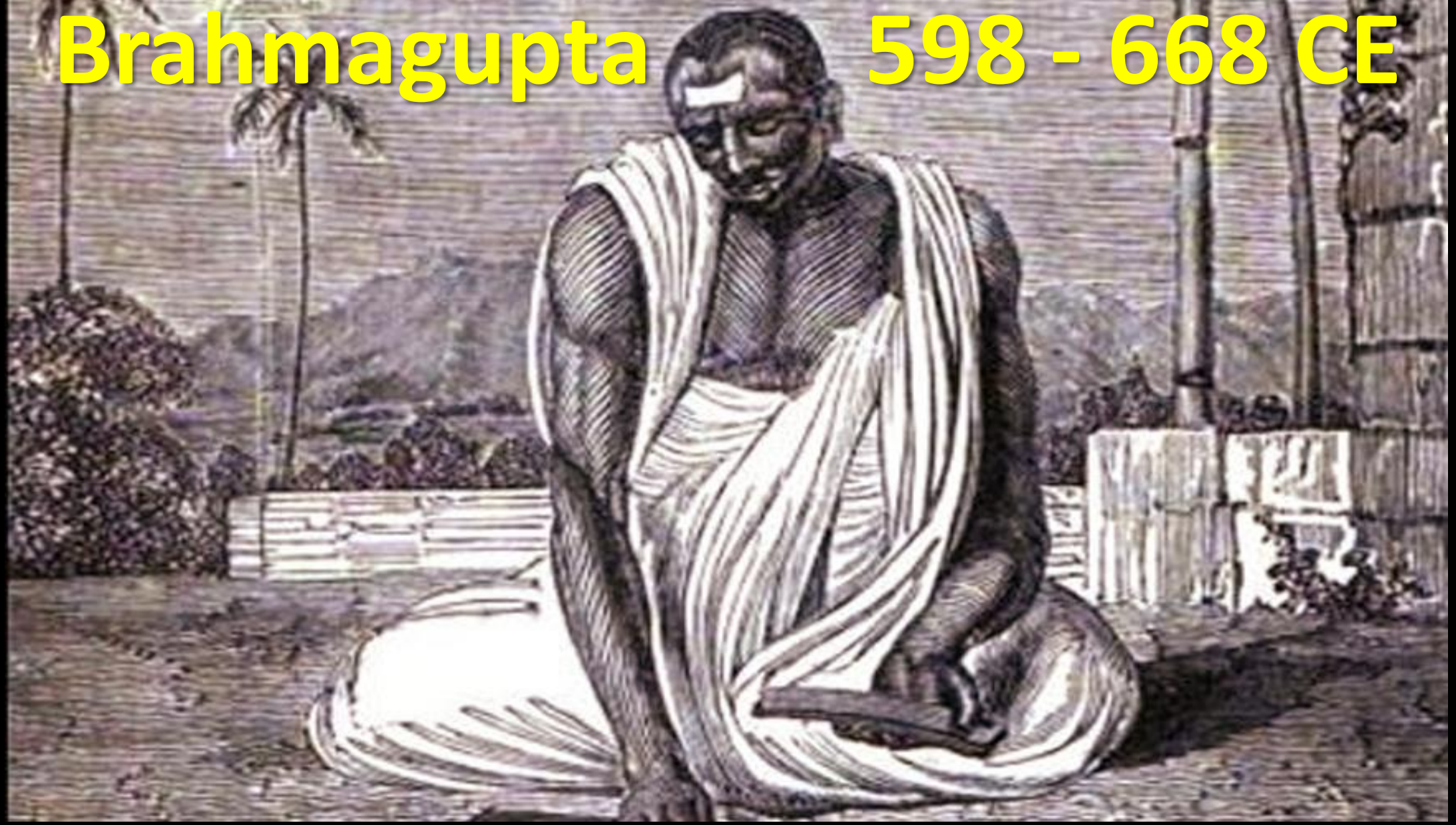
Extraordinary Evidence...

© 2019 J. J. Crabtree | www.jonathancrabtree.com


Brahmagupta 598 - 668 CE



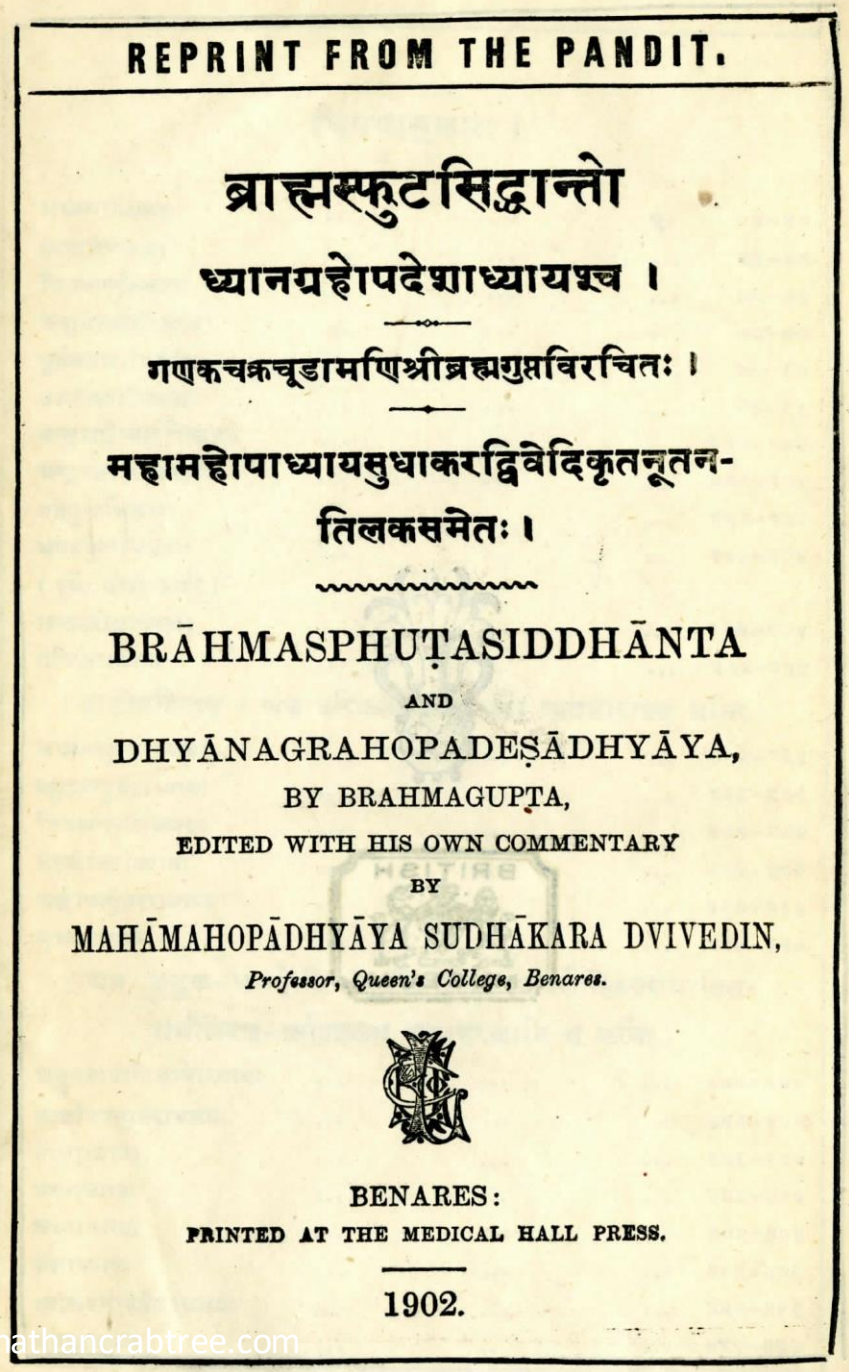
Astronomer and Mathematician



Brāhmasphuṭasiddhānta 628 CE

For this talk, Brahmagupta's Laws of Positives Negatives and Zero have been freshly analysed.

Images courtesy of the British Library.



REPRINT FROM THE PANDIT.

ब्राह्मस्फुटसिद्धान्तो

ध्यानग्रहोपदेशाध्यायश्च ।

गणकचक्रचूडामणिश्रीब्रह्मगुप्तविरचितः ।

महामहोपाध्यायसुधाकरद्विवेदिकृतनूतन-तिलकसमेतः ।

BRĀHMASPHUṬASIDDHĀNTA

AND

DHYĀNAGRAHOPADEṢĀDHYĀYA,

BY BRAHMAGUPTA,

EDITED WITH HIS OWN COMMENTARY

BY

MAHĀMAHOPĀDHYĀYA SUDHĀKARA DVIVEDIN,

Professor, Queen's College, Benares.

BENARES:

PRINTED AT THE MEDICAL HALL PRESS.

1902.

Brahmagupta's 5 Addition Sutras

SL

(saṅkalana)

© 2020 Jonathan J. Crabtree | www.podometic.in

Brahmagupta's 5 Addition Sutras

AS1 **positive** plus **positive** is **positive**

AS2 **negative** plus **negative** is **negative**

AS3 **positive** plus **negative** is the difference between the **positive** and **negative**

AS4 when **positive** and **negative** are equal the sum is **zero**

AS5
positive plus **zero** is **positive**
negative plus **zero** is **negative**
zero plus **zero** is **zero**

© 2020 Jonathan J. Crabtree | www.podometic.in

Brahmagupta's 5 Addition Sutras

AS1 **positive** plus **positive** is **positive**

AS2 **negative** plus **negative** is **negative**

AS3 **positive** plus **negative** is the difference between the **positive** and **negative**

AS4 when **positive** and **negative** are equal the sum is **zero**

AS5
positive plus **zero** is **positive**
negative plus **zero** is **negative**
zero plus **zero** is **zero**

© 2020 Jonathan J. Crabtree | www.podometic.in

Brahmagupta's 5 Addition Sutras

AS1 **positive** plus **positive** is **positive** **Understood in Arabic world**

AS2 **negative** plus **negative** is **negative**

AS3 **positive** plus **negative** is the difference between the **positive** and **negative**

AS4 when **positive** and **negative** are equal the sum is **zero**

AS5
positive plus **zero** is **positive**
negative plus **zero** is **negative**
zero plus **zero** is **zero**

© 2020 Jonathan J. Crabtree | www.podometic.in

Al-Khwārizmī (c. 780-850)

I had seen that the Indians had set up **9 symbols** in their universal system of numbering…

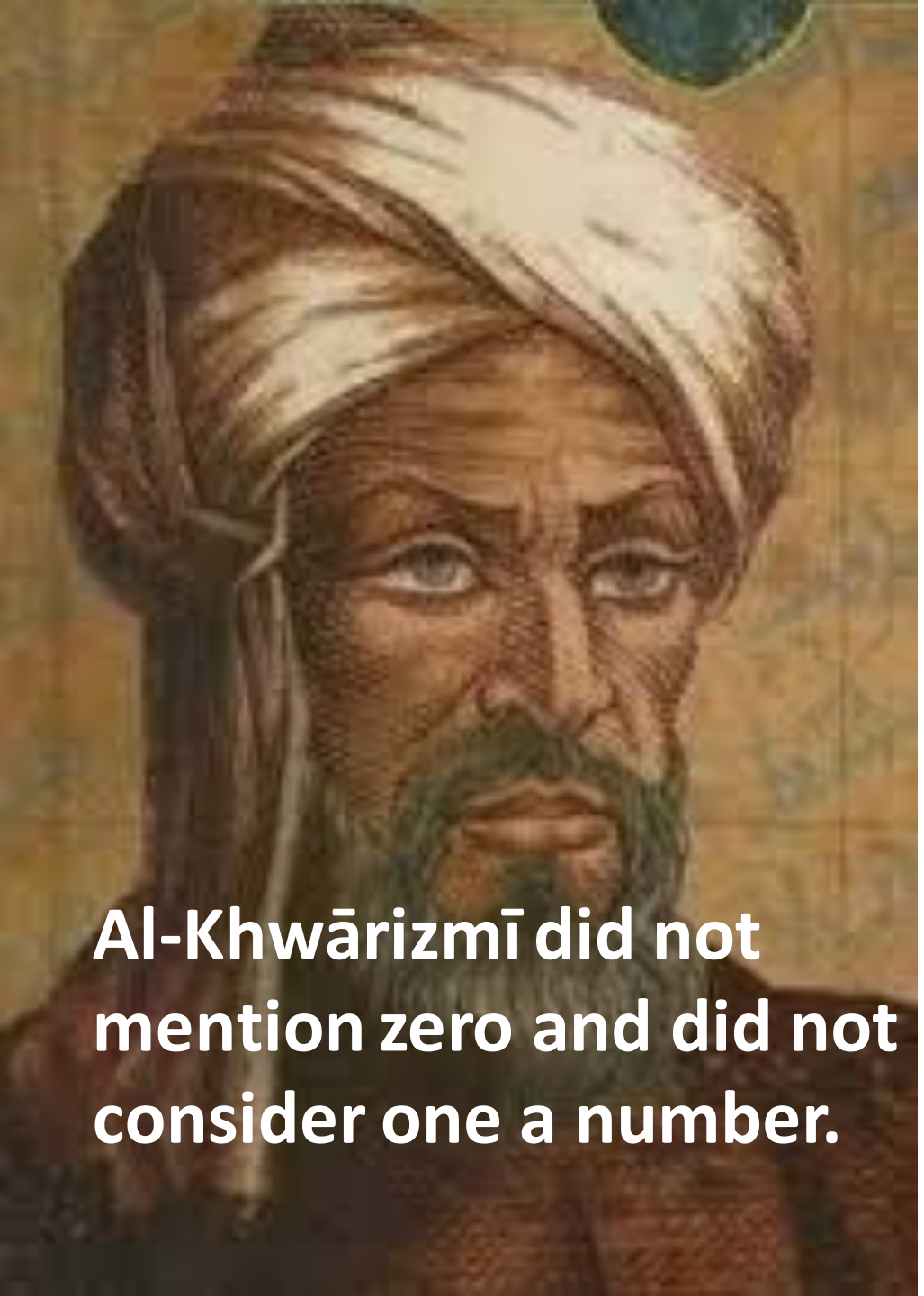
Al-Khwārizmī did not mention zero

Al-Khwārizmī (c. 780-850)

So they made **9 symbols**, which are these: 9 8 7 6 5 4 3 2 1.

And … **every number is put together above one**.

"Algorizmi said: since I had seen that the Indians had set up IX symbols..." Crossley, John N, and Henry, Alan S. (1990) *Thus Spake Al-Ḵẖwārizmī: A Translation of the Text of Cambridge University Library Ms. Ii. Vi. 5*. Historia Mathematica. P. 110-111



Al-Khwārizmī did not mention zero and did not consider one a number.

Al-Khwārizmī (c. 780-850)

… **one** is the root of all number and **is outside number**.

It is the root of number because every number is found by it.

But it [one] is outside number because it is found by itself, I mean, without any other number.

Al-Uqlidisi (c. 920-980)

Al-Uqlidisi considered zero a placeholder, not a number.

Al-Uqlidisi (c. 920-980)

Al-Uqlidisi considered zero an empty place-holder, not a number.

One question is: **Why are the Hindi letters nine**, no more, no less? We say: Because the beginning of numbers from which they start is one and the last unit we pronounce is nine. Thus when we say units we mean something between one and nine; after that units are over, and ten comes out like one and takes its form. We add up ten to ten until we reach 90 which conforms with nine. Tens are now over and we say one hundred, coming back to one, and going up to 9. Thus we see that all places start with one and end with nine. That is why they are made nine. **So much for the nine letters.** If it is said: Why is zero multiplied by zero equal to zero and zero multiplied by any letter zero? We say that by multiplying zero by **zero the aim is only to occupy the place**; the same applies for multiplying the letter by zero. **We multiply the letter by zero** only once, the first time, by the first letter in the upper, **to occupy the place**, and **tell that there is a place and that it is empty.**

Saidan, Ahmad S. (1978) *The Arithmetic of Al-Uqlídisí: The Story of Hindu-Arabic Arithmetic As Told in Kitab Al-Fusul Fi Al-Hisab Al-Hindi.* Reidel, Dordrecht. P. 186

200 years after Brahmagupta, al-Khwārizmī did not accept 1 as a number. Zero as a number? Never!

200 years after Brahmagupta, al-Khwārizmī did not accept 1 as a number. Zero as a number? Never!

300 years after Brahmagupta, al-Uqlīdisī accepted India's **ZERO** as a placeholder, yet not a number. Why?

© 2019 J. J. Crabtree | www.jonathancrabtree.com

Al-Uqlīdisī means ‘the Euclidist’. He was known for his skill in studying the Greek geometry of Euclid and translating it into Arabic.

© 2019 J. J. Crabtree | www.jonathancrabtree.com

Al-Uqlīdisī means ‘the Euclidist’. He was known for his skill in studying the Greek geometry of Euclid and translating it into Arabic.

Around 300 BCE, Euclid defined ‘number’ as *a multitude of units*. So Euclid’s definition of number came before 0 and 1 were numbers.

© 2019 J. J. Crabtree | www.jonathancrabtree.com

India defined zero as the sum of opposing negative and positive numbers or quantities with the same multitude or magnitude.

India defined zero as the sum of opposing negative and positive numbers / quantities with the same multitude or magnitude.

If Arabic and European writers in medieval times *really* understood India's zero, where are all the negative numbers in their writings?

© 2019 J. J. Crabtree | www.jonathancrabtree.com

“I have read a few dozen medieval Arabic books on arithmetic and algebra, and there is no hint of negative numbers in any of them. Zero, too, was not regarded to be a number, but was merely the place holder for an empty place in the representation of a number in Arabic (Indian) notation.”

By email courtesy of Dr. Jeffrey Oaks, Professor of Mathematics

Medieval Arabic algebra and the mathematics of Greece and medieval Europe UINDY

"I have read a few dozen medieval Arabic books on arithmetic and algebra, and there is no hint of negative numbers in any of them. Zero, too, was not regarded to be a number, but was merely the place holder for an empty place in the representation of a number in Arabic (Indian) notation."

"All numbers in Arabic arithmetic were positive. No Arabic author to my knowledge ever even contemplated the existence of negative numbers."

By email courtesy of Dr. Jeffrey Oaks, Professor of Mathematics
Medieval Arabic algebra and the mathematics of Greece and medieval Europe UINDY

The maths MYTHS we know

India’s definition of zero as a number that was a sum of equal, yet opposite negative and positive quantities was **[NOT]** embraced by the Arabic world

The maths MYTHS we know

India's definition of zero as a number that was a sum of equal, yet opposite negative and positive quantities was [NOT] embraced by the Arabic world

Al-Khwārizmī wrote a book on Hindu Integer arithmetic which **[DID NOT]** feature Brahmagupta's ancient laws of sign for negatives and positives

The maths MYTHS we know

Based on what he learned and documented in his book Algoritmi de numero Indorum (al-Khwārizmī on the Hindu Art of Reckoning) he then wrote a book on algebra **[NO]**

Al-Khwārizmī wrote his book on algebra BEFORE he understood Indian mathematics.

The maths MYTHS we know

Al-Khwārizmī did **NOT** remove negative terms from his equations.

The maths MYTHS we know

Al-Khwārizmī did **NOT** remove negative terms from his equations.

Al-Khwārizmī simply eliminated any positive term that was being subtracted in an equation.

The maths MYTHS we know

Al-Khwārizmī did **NOT** remove negative terms from his equations.

Al-Khwārizmī simply eliminated any positive term that was being subtracted in an equation.

For example, $ax^2 = bx - c$ became $ax^2 + c = bx$.

The maths MYTHS we know

Comparing al-Khwārizmī's approach to Brahmagupta's,

Rashed, Roshdi. (2009) *The Beginnings of Algebra*. Saqi, London.

The maths MYTHS we know

Comparing al-Khwārizmī's approach to Brahmagupta's

Rashed, Roshdi. (2009) *The Beginnings of Algebra*. Saqi, London.

"Once again al-Khwārizmī differs from Brahmagupta, this time in not employing any abbreviation.

The maths MYTHS we know

Comparing al-Khwārizmī's approach to Brahmagupta's

Rashed, Roshdi. (2009) *The Beginnings of Algebra*. Saqi, London.

Al-Khwārizmī… avoids using "negative" numbers or simply a [larger] number subtracted from a smaller one, or from zero, whereas Brahmagupta, like other Indian mathematicians before him, does not hesitate to make use of such [negative] numbers."

The maths MYTHS we know

"It is difficult to imagine that al-Khwārizmī, if he had read this chapter [i.e. chapter 18 of Brahmagupta's Brāhma Sphutasiddhānta] would not have been able to profit by it, even if only to shorten the presentation of his work."

The maths MYTHS we know

"It is difficult to imagine that al-Khwārizmī, if he had read this chapter [i.e. chapter 18 of Brahmagupta's Brāhma Sphutasiddhānta] would not have been able to profit by it, even if only to shorten the presentation of his work."

"The style of the mathematical reasoning that is at work in al-Khwārizmī's algebra has nothing to do with what we encounter in the work of his (Indian) predecessors."

628 CE Brahmagupta had everything we need today!

© 2019 J. J. Crabtree | www.jonathancrabtree.com

628 CE
Brahmagupta had everything we need today!

830 CE
Al-Khwārizmī did not have 1 as a number.

© 2019 J. J. Crabtree | www.jonathancrabtree.com

628 CE
Brahmagupta had everything we need today!

830 CE
Al-Khwārizmī did not have 1 as a number.

950 CE
Al-Uqlīdisī only had 0 as a placeholder

Teach Brahmagupta's Better Bharatan Maths! Sign the Petition at www.j.mp/BharatanMaths

© 2019 J. J. Crabtree | www.jonathancrabtree.com

628 CE
Brahmagupta had everything we need today!

830CE
Al-Khwārizmī did not have 1 as a number.

950 CE
Al-Uqlīdisī only had 0 as a placeholder

Leonardo Pisano
1170 – 1250

© 2019 J. J. Crabtree | www.jonathancrabtree.com

628 CE
Brahmagupta had everything we need today!

830CE
Al-Khwārizmī did not have 1 as a number.

950 CE
Al-Uqlīdisī only had 0 as a placeholder

I am Leonardo Pisano. I am the man most responsible for introducing India's arithmetic into Europe in the 13th Century via my book Liber Abaci.

Leonardo Pisano

© 2019 J. J. Crabtree | www.jonathancrabtree.com

628 CE Brahmagupta had everything we need today!

830CE
Al-Khwārizmī
did not have 1
as a number.

950 CE
Al-Uqlīdisī
only had 0 as
a placeholder

As I got my Indian info via Arabic traders, I did **NOT** get to learn about India's definition of zero as a number or the rules of positive and negatives.

© 2019 J. J. Crabtree | www.jonathancrabtree.com

Teach Brahmagupta's Better Bharatan Maths! Sign the Petition at www.j.mp/BharatanMaths

628 CE Brahmagupta had everything we need today!

830CE
Al-Khwārizmī
did not have 1
as a number.

950 CE
Al-Uqlīdisī
only had 0 as
a placeholder

As I got my Indian info via Arabic traders, I did **NOT** get to learn about India's definition of zero as a number or the rules of positive and negatives. **Whoops!**

© 2019 J. J. Crabtree | www.jonathancrabtree.com

Teach Brahmagupta's Better Bharatan Maths! Sign the Petition at www.j.mp/BharatanMaths

Map data © 2019 Google
Transmission of zero as a placeholder, yet not as defined by Brahmagupta 628 CE.
© 2019 J. J. Crabtree
www.jonathancrabtree.com
Pisa
Italy
Béjaïa
Baghdad
Bhinmal
United Kingdom
Ireland
France
Spain
Portugal
Germany
Poland
Belarus
Ukraine
Kazakhstan
Uzbekistan
Turkey
Greece
Iraq
Iran
Saudi Arabia
India
Nepal
Bhutan
Bangladesh
Sri Lanka
Yemen
Eritrea
Ethiopia
Nigeria
Tunisia
Morocco
Mediterranean Sea
Black Sea
Caspian Sea
Red Sea
Persian Gulf
Arabian Sea
Bay of Bengal

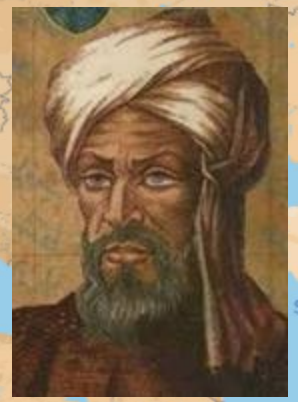
Map data © 2019 Google
Transmission of zero as a placeholder, yet not as defined by Brahmagupta 628 CE.
© 2019 J. J. Crabtree
www.jonathancrabtree.com
Pisa
Italy
Béjaïa
Baghdad
Bhinmal
Iraq
Iran
Syria
Lebanon
Israel
Jordan
Turkey
Greece
Bulgaria
Romania
Serbia
Hungary
Austria
Slovakia
Czechia
Germany
France
Belgium
Netherlands
Poland
Belarus
Ukraine
Moldova
Lithuania
Latvia
Estonia
Denmark
United Kingdom
Ireland
Portugal
Spain
Morocco
Tunisia
Libya
Egypt
Saudi Arabia
Yemen
Oman
United Arab Emirates
Kazakhstan
Uzbekistan
Turkmenistan
India
Nepal
Bhutan
Bangladesh
Sri Lanka
Ethiopia
Eritrea
Djibouti
Nigeria
Cameroon
South Sudan
Central African Republic
Mediterranean Sea
Black Sea
Red Sea
Persian Gulf
Gulf of Aden
Arabian Sea
Bay of Bengal
Moscow
Novosibirsk
Astana
Rome
Barcelona
Madrid
Istanbul
Riyadh
Mecca
Mumbai
Bengaluru
Lagos

Transmission of zero as a placeholder, yet not as defined by Brahmagupta 628 CE.

Map data © 2019 Google

© 2019 J. J. Crabtree
www.jonathancrabtree.com

Map data © 2019 Google
United Kingdom
Ireland
Pisa
Italy
Rome
Béjaïa
Baghdad
Iraq
Iran
Bhinmal
Transmission of zero as a placeholder, yet not as defined by Brahmagupta 628 CE.
© 2019 J. J. Crabtree
www.jonathancrabtree.com



Map data © 2019 Google

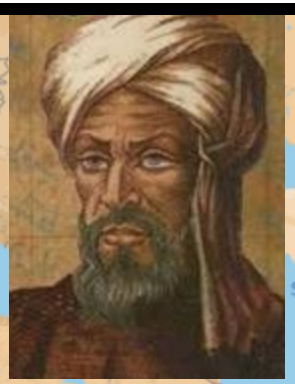
Transmission of zero as a placeholder, yet not as defined by Brahmagupta 628 CE.

Those unaware zero was defined as the sum of equal pos. & neg.
- Al-Khwārizmī, Iraq 9th C.
- Traders, Nth Africa 12th C.
- Leonardo Pisano, Italy 13th C.
- Robert Recorde, England 16th C.

© 2019 J. J. Crabtree
www.jonathancrabtree.com

I invented the sign = and introduced the pre-existing sign + to England.

© 2019 Google



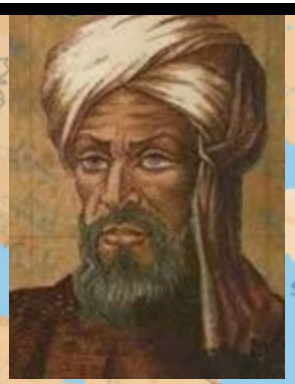
Transmission of zero as a placeholder, yet not as defined by Brahmagupta 628 CE.

Those unaware zero was defined as the sum of equal pos. & neg.
- Al-Khwārizmī, Iraq 9th C.
- Traders, Nth Africa 12th C.
- Leonardo Pisano, Italy 13th C.
- Robert Recorde, England 16th C.

© 2019 J. J. Crabtree
www.jonathancrabtree.com
Map Data © 2019 Google

I never knew your 0 definition Brahmagupta or your 18 sutras for 0, positives & negatives!

Recorde knew a subtraction subtracted is like an addition yet that's another idea.



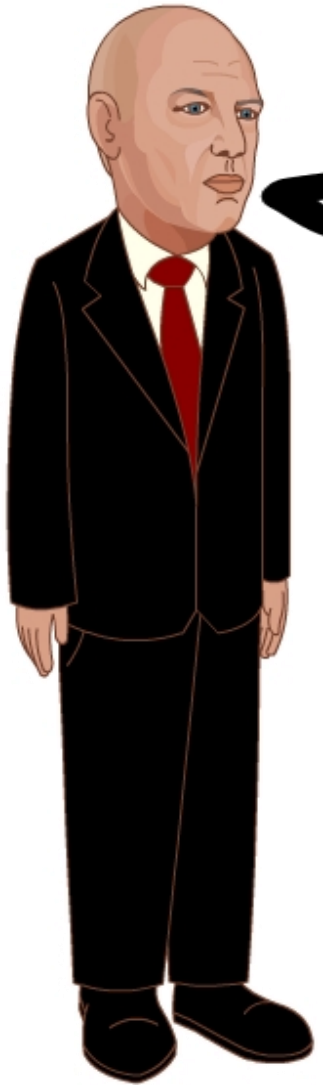
Transmission of zero as a placeholder, yet not as defined by Brahmagupta 628 CE.

Those unaware zero was defined as the sum of equal pos. & neg.

- Al-Khwārizmī, Iraq 9th C.
- Traders, Nth Africa 12th C.
- Leonardo Pisano, Italy 13th C.
- Robert Recorde, England 16th C.

© 2020 Jonathan J. Crabtree
www.podometic.in
Map Data © 2019 Google

Copyright © 2019
All Rights Reserved
Jonathan J. Crabtree



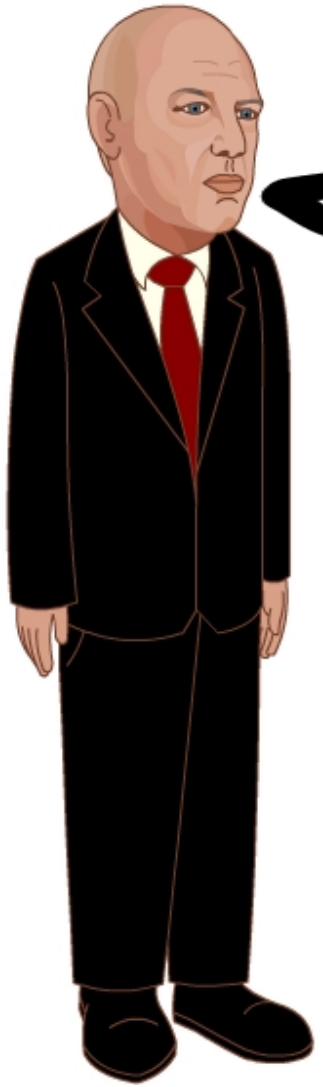
In 1478 the first book printed on maths (Treviso Arithmetic) said numbers start at 2.

So much for 0 and 1 which is all your computer needs!

Sign the Petition!

www.j.mp/TeachIndianMaths

Copyright © 2019
All Rights Reserved
Jonathan J. Crabtree

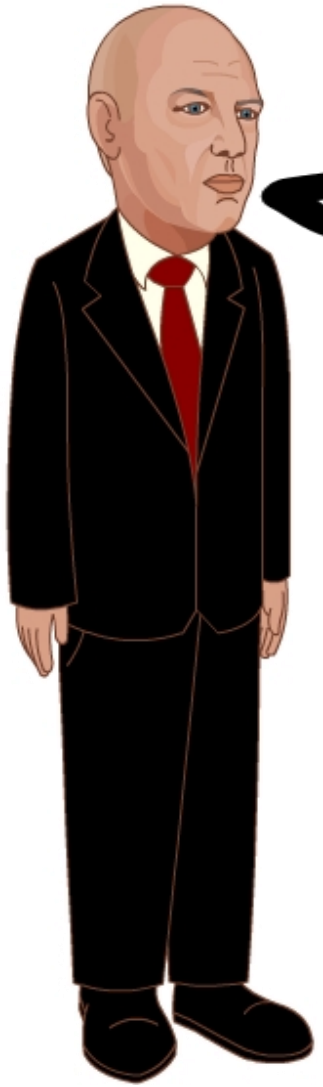


In 16th Century England people used Roman Numerals and there was no Roman Numeral for 0.

Sign the Petition!

www.j.mp/TeachIndianMaths

Copyright © 2019
All Rights Reserved
Jonathan J. Crabtree



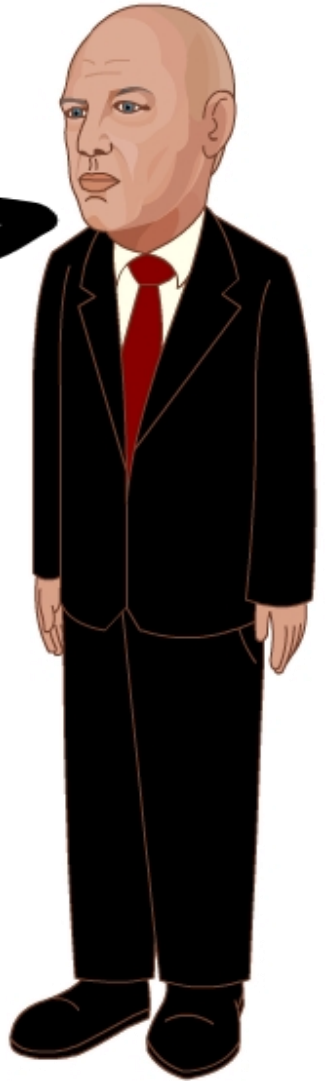
The maths at the time was based on Ancient Greek maths which did not have zero, one or negative numbers.

Sign the Petition!

www.j.mp/TeachIndianMaths

Copyright © 2019
All Rights Reserved
Jonathan J. Crabtree

The false idea negative quantities are less than zero (rather than opposite in nature to positive quantities) emerged in Michael Stiefel's *Arithmetica Integra* of 1544 in a section titled De signis additorum & subtractorum & de numeris absurdis.

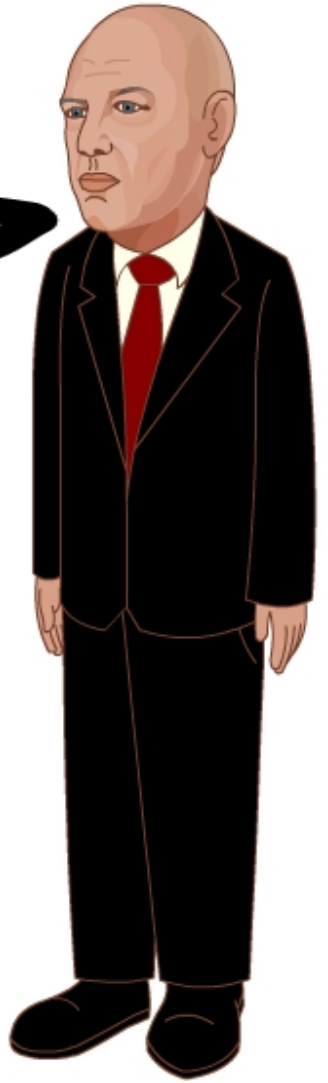


Sign the Petition!

www.j.mp/TeachIndianMaths

Copyright © 2019
All Rights Reserved
Jonathan J. Crabtree

Michael Stiefel said negative numbers were below zero which is below nothing *infra 0, id est infra nihil* **and negative numbers were absurd** *numeri absurdi*.

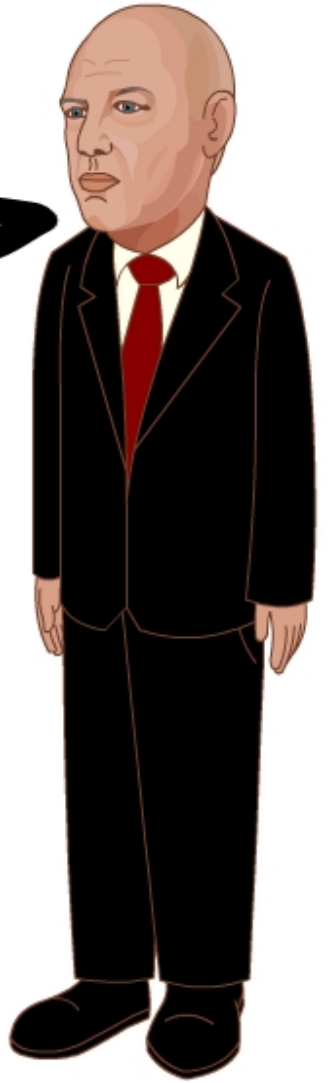


Sign the Petition!

www.j.mp/TeachIndianMaths

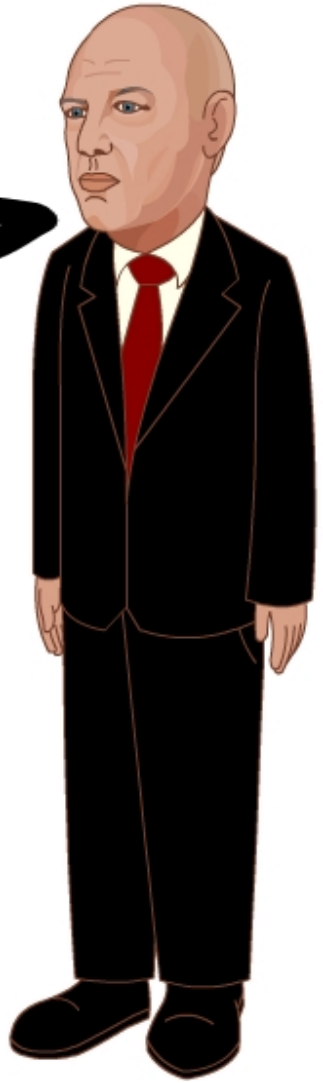
Copyright © 2019
All Rights Reserved
Jonathan J. Crabtree

To make sense of numbers that count or measure negative quantities, (i.e. negative numbers) all we need to do is drop the nonsense notion that negative quantities are 'less than zero'.



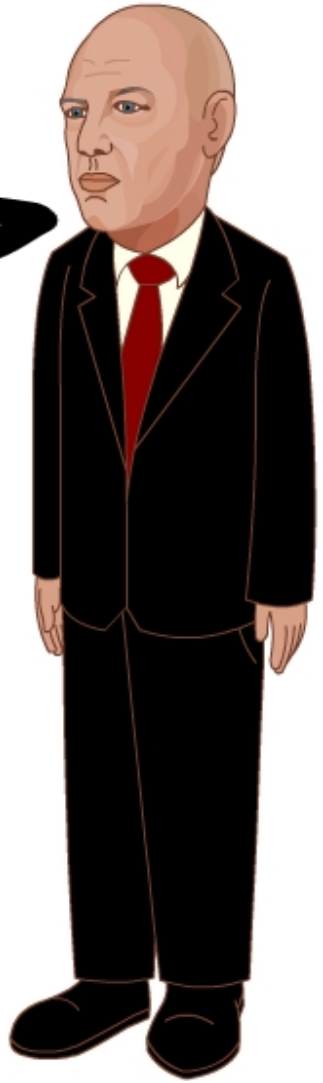
Copyright © 2019
All Rights Reserved
Jonathan J. Crabtree

Then negative numbers simply count or measure opposite quantities or forces, which are always greater than zero!



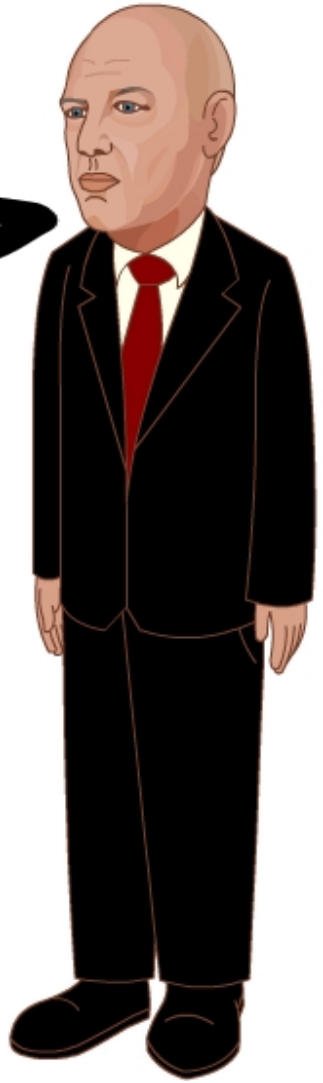
Copyright © 2019
All Rights Reserved
Jonathan J. Crabtree

Perhaps you might recall Newton's Third Law which states for every action there is an equal and opposite reaction. Bingo!



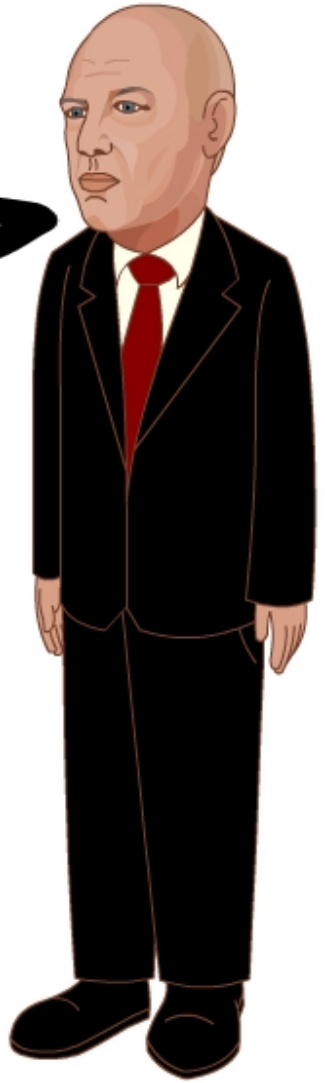
Copyright © 2019
All Rights Reserved
Jonathan J. Crabtree

Newton's Laws of Motion are consistent with Brahmagupta's laws of quantitative mathematics, which are also consistent with quantum physics.

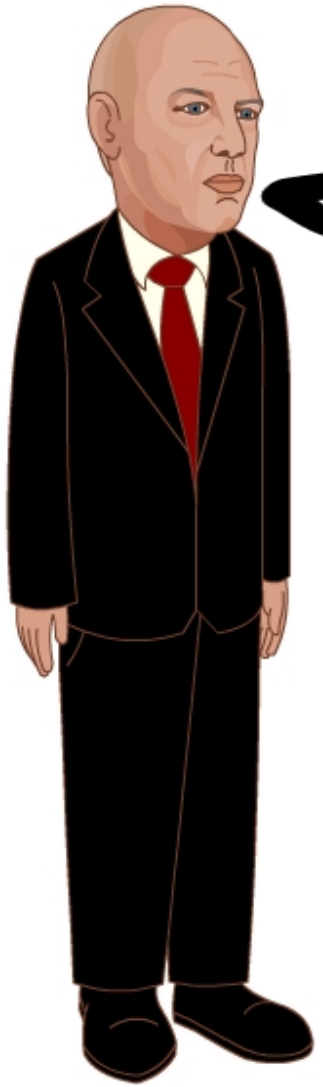


Copyright © 2019
All Rights Reserved
Jonathan J. Crabtree

Think about it... 3 negative electrons and 3 positive positrons cancel each other out to sum to zero.

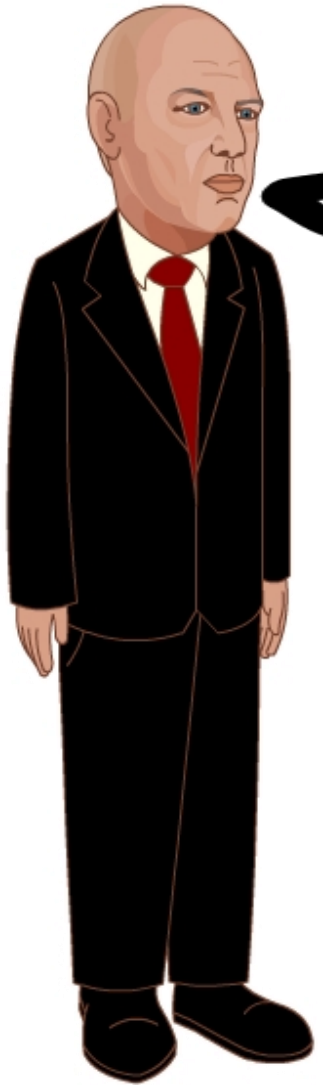


Copyright © 2019
All Rights Reserved
Jonathan J. Crabtree



As maths books got published in the English language, (without 0 or 1 in algorithmic definitions or as numbers) they were exported to England's settlements and colonies (e.g. New England became America).

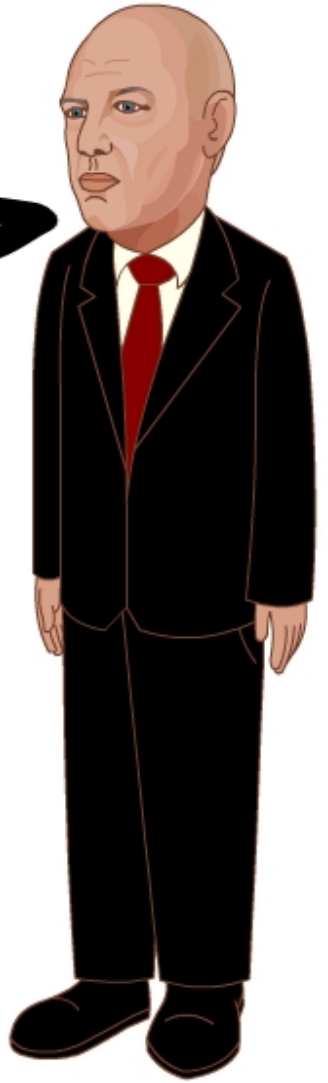
Copyright © 2019
All Rights Reserved
Jonathan J. Crabtree



So as the English language spread, so too did **major misunderstandings** of India's mathematical foundations!

Copyright © 2019
All Rights Reserved
Jonathan J. Crabtree

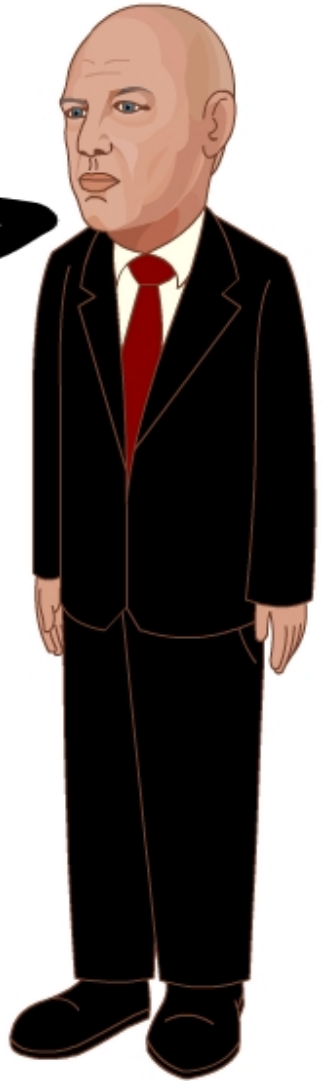
In 628 Brahmagupta gave solutions to equations we'd write today as $x^2 - 92y^2 = 1$* and $ax^2 + bx + c = 0$.



Copyright © 2019
All Rights Reserved
Jonathan J. Crabtree

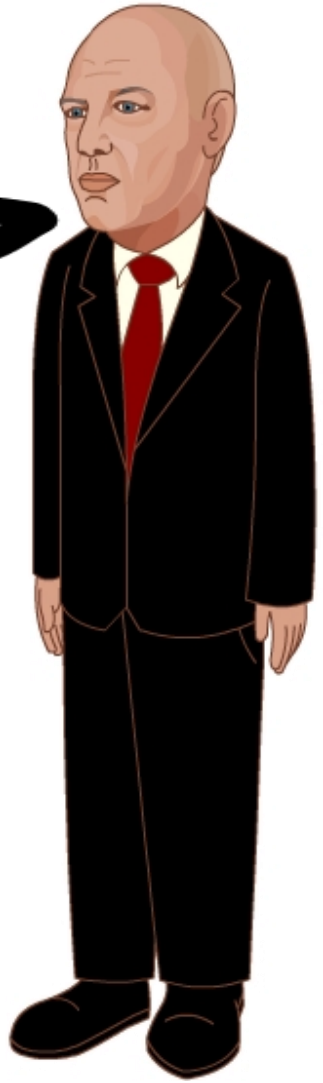
In 628 Brahmagupta gave solutions to equations we'd write today as $x^2 - 92y^2 = 1$*

$(x = 1151 \text{ and } y = 120)$



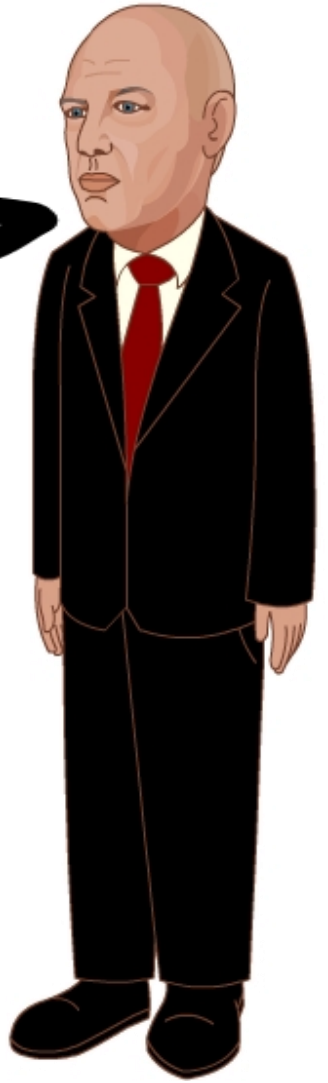
Copyright © 2019
All Rights Reserved
Jonathan J. Crabtree

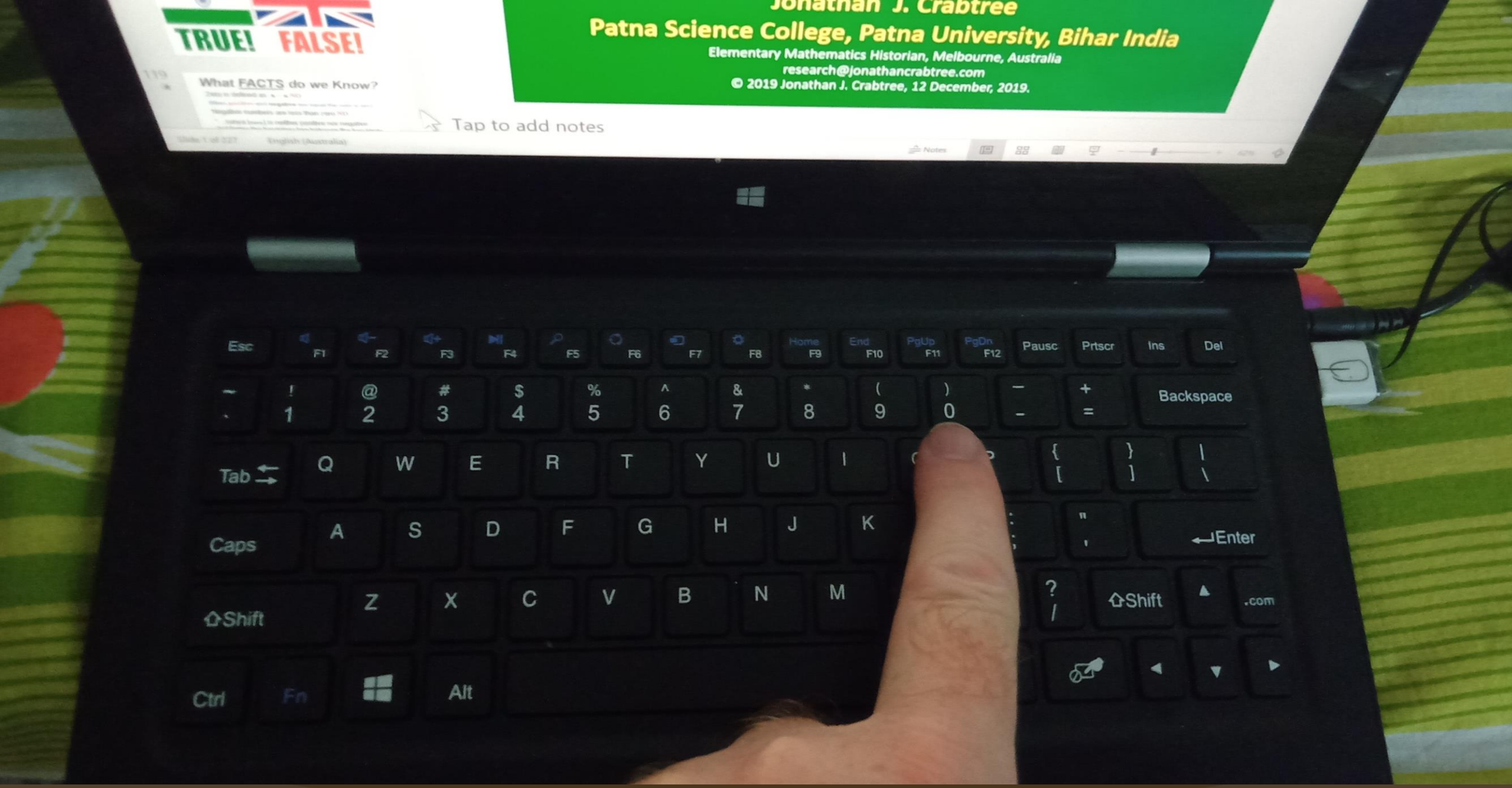
However, the first person to say 1 was a number in the West was Simon Stevin in 1585, almost 1000 years after Brahmagupta!



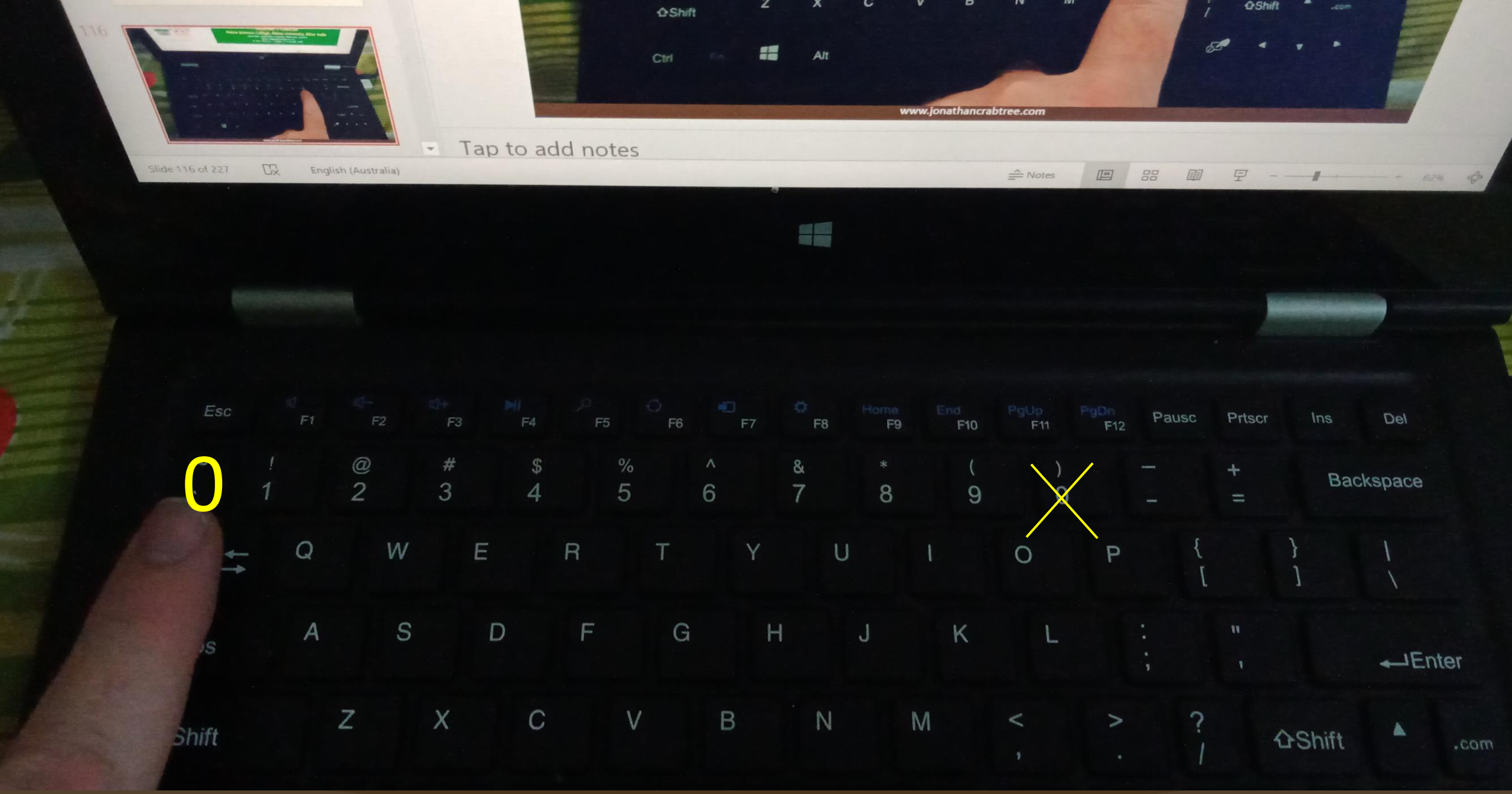
Copyright © 2019
All Rights Reserved
Jonathan J. Crabtree

However, the first person to say 1 was a number in the West was Simon Stevin in 1585, almost 1000 years after Brahmagupta! **So, where is zero today?**





© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com



© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

Brahmagupta's 5 Addition Sutras

धनयोर्धनम्ऋणमृणयोः धनर्णयोरन्तरं समैक्यं खम् ऋणमैक्यं च धनमृणधनशून्ययोः शून्ययोः शून्यम्

AS1 **positive** plus **positive** is **positive**

AS2 **negative** plus **negative** is **negative**

AS3 **positive** plus **negative** is the difference between the **positive** and **negative**

AS4 when **positive** and **negative** are equal the sum is **zero**

AS5
positive plus **zero** is **positive**
negative plus **zero** is **negative**
zero plus **zero** is **zero**

© 2020 Jonathan. J. Crabtree | www.podometic.in | Sign the Petition for Better Bharatiya Maths @ www.j.mp/BharatiyaMaths

Brahmagupta's 5 Subtraction Sutras

ऊनमधिकाद्विशोध्यं धनं धनाद्ऋणमृणाद्अधिकमूनात् व्यस्तं तदन्तरं स्यादृणं धनं धनमृणं भवति
शून्यविहीनमृणमृणं धनं धनं भवति शून्यमाकाशम् शोध्यं यदा धनमृणाद्ऋणं धनाद्वा तदा क्षेप्यम

SS1 A smaller **positive** subtracted from a larger **positive** is **positive**.

SS2 A smaller **negative** subtracted from a larger **negative** is **negative**.

SS3 If a larger **negative** or **positive** is to be subtracted from a smaller **negative** or **positive**, the sign of their difference is reversed – **negative** becomes **positive** and **positive** **negative**.

SS4 A **negative** minus **zero** is **negative**,
a **positive** minus **zero** is **positive**,
zero minus **zero** is **zero**.

SS5 When a **positive** is to be subtracted from a **negative** or a **negative** from a **positive**, then it is to be added.

© 2020 Jonathan. J. Crabtree | www.podometic.in | Sign the Petition for Better Bharatiya Maths @ www.j.mp/BharatiyaMaths

Brahmagupta's 4 Multiplication Sutras

ऋणमृणधनयोर्घातो धनमृणयोः धनवधो धनं भवति
शून्यर्णयोः खधनयोः खशून्ययोर्वा वधः शून्यम्

MS1 The product of a **negative** and a **positive** is **negative**.

MS2 The product of two **negatives** is **positive**.

MS3 The product of two **positives** is **positive**.

MS4 The product of **zero** and a **negative**,
of **zero** and a **positive**, or
of two **zeros** is **zero**.

© 2020 Jonathan. J. Crabtree | www.podometic.in | Sign the Petition for Better Bharatiya Maths @ www.j.mp/BharatiyaMaths

Brahmagupta's 4 Division Sutras

धनभक्तं धनम् ऋणहृतमृणं धनं भवति खं खभक्तं खम्
भक्तमृणेन धनमृणं धनेन हृतम् ऋणमृणं भवति
खोद्धृतमृणं धनं वा तच्छेदं खमृणधनविभक्तं वा
ऋणधनयोर्वर्गः स्वं खं खस्य पदं कृतिर्यत् तत्

DS1 A **positive** divided by a **positive** is **positive**.

DS2 A **negative** divided by a **negative** is **positive**.

DS3 A **positive** divided by a **negative** is **negative**.

DS4 A **negative** divided by a **positive** is **negative**.

Acknowledgement: I am grateful to Avinash Sathaye, K. Ramasubramanian, Clemency Montelle, Kim Plofker and Agathe Keller. Analysis, interpretation *(& any mistakes)* by Jonathan J. Crabtree.

© 2020 Jonathan. J. Crabtree | www.podometic.in | Sign the Petition for Better Bharatiya Maths @ www.j.mp/BharatiyaMaths

Brahmagupta's 5 Addition Sutras

धनयोर्धनम्ऋणमृणयोः धनर्णयोरन्तरं समैक्यं खम् ऋणमैक्यं च धनमृणधनशून्ययोः शून्ययोः शून्यम्

AS1 positive plus positive is positive

AS2 negative plus negative is negative

AS3 positive plus negative is the difference between the positive and negative

AS4 when positive and negative are equal the sum is zero

AS5
positive plus zero is positive
negative plus zero is negative
zero plus zero is zero

© 2020 Jonathan. J. Crabtree | www.podometic.in | Sign the Petition for Better Bharatiya Maths @ www.j.mp/BharatiyaMaths

Brahmagupta's 5 Subtraction Sutras

ऊनमधिकाद्विशोध्यं धनं धनाद्ऋणमृणाद्अधिकमूनात् व्यस्तं तदन्तरं स्याद्दृणं धनं धनमृणं भवति
शून्यविहीनमृणमृणं धनं धनं भवति शून्यमाकाशम् शोध्यं यदा धनमृणाद्ऋणं धनाद्वा तदा क्षेप्यम

SS1 A smaller positive subtracted from a larger positive is positive.

SS2 A smaller negative subtracted from a larger negative is negative.

SS3 If a larger negative or positive is to be subtracted from a smaller negative or positive, the sign of their difference is reversed – negative becomes positive and positive negative.

SS4
A negative minus zero is negative,
a positive minus zero is positive,
zero minus zero is zero.

SS5 When a positive is to be subtracted from a negative or a negative from a positive, then it is to be added.

© 2020 Jonathan. J. Crabtree | www.podometic.in | Sign the Petition for Better Bharatiya Maths @ www.j.mp/BharatiyaMaths

Brahmagupta's 4 Multiplication Sutras

ऋणमृणधनयोर्घातो धनमृणयोः धनवधो धनं भवति
शून्यर्णयोः खधनयोः खशून्ययोर्वा वधः शून्यम्

MS1 The product of a negative and a positive is negative.

MS2 The product of two negatives is positive.

MS3 The product of two positives is positive.

MS4
The product of zero and a negative,
of zero and a positive, or
of two zeros is zero.

© 2020 Jonathan. J. Crabtree | www.podometic.in | Sign the Petition for Better Bharatiya Maths @ www.j.mp/BharatiyaMaths

Brahmagupta's 4 Division Sutras

धनभक्तं धनम् ऋणहृतमृणं धनं भवति खं खभक्तं खम्
भक्तमृणेन धनमृणं धनेन हृतम् ऋणमृणं भवति
खोद्धृतमृणं धनं वा तच्छेदं खमृणधनविभक्तं वा
ऋणधनयोर्वर्गः स्वं खं खस्य पदं कृतिर्यत् तत्

DS1 A positive divided by a positive is positive.

DS2 A negative divided by a negative is positive.

DS3 A positive divided by a negative is negative.

DS4 A negative divided by a positive is negative.

Acknowledgement: I am grateful to Avinash Sathaye, K. Ramasubramanian, Clemency Montelle, Kim Plofker and Agathe Keller. Analysis, interpretation *(& any mistakes)* by Jonathan J. Crabtree.

© 2020 Jonathan. J. Crabtree | www.podometic.in | Sign the Petition for Better Bharatiya Maths @ www.j.mp/BharatiyaMaths

When positive and negative are equal the sum is zero

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

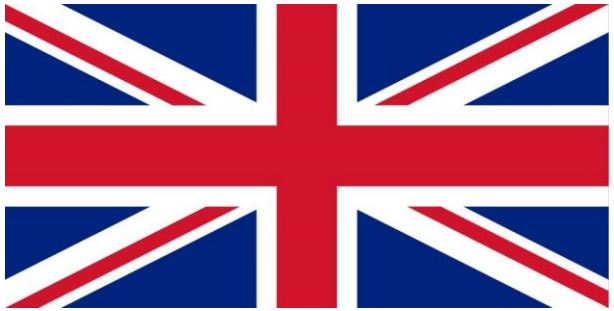
When positive and negative are equal the sum is zero

Correct according to the laws of physics, yet NOT what we are taught in school!

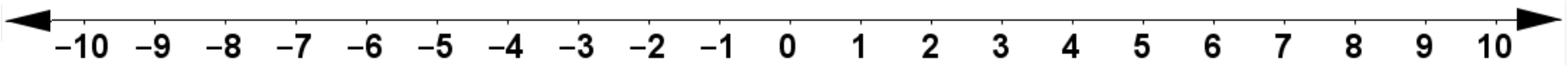
© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

When positive and negative are equal the sum is zero

Correct according to the laws of physics, yet NOT what we are taught in school!



Negative Integers are less than Positive Integers.

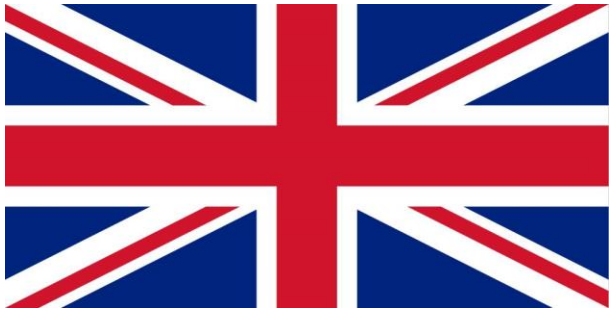


The more left you go, the smaller the number. **NO!**

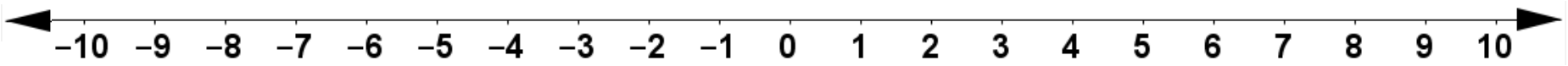
© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

When positive and negative are equal the sum is zero

Correct according to the laws of physics, yet NOT what we are taught in school!



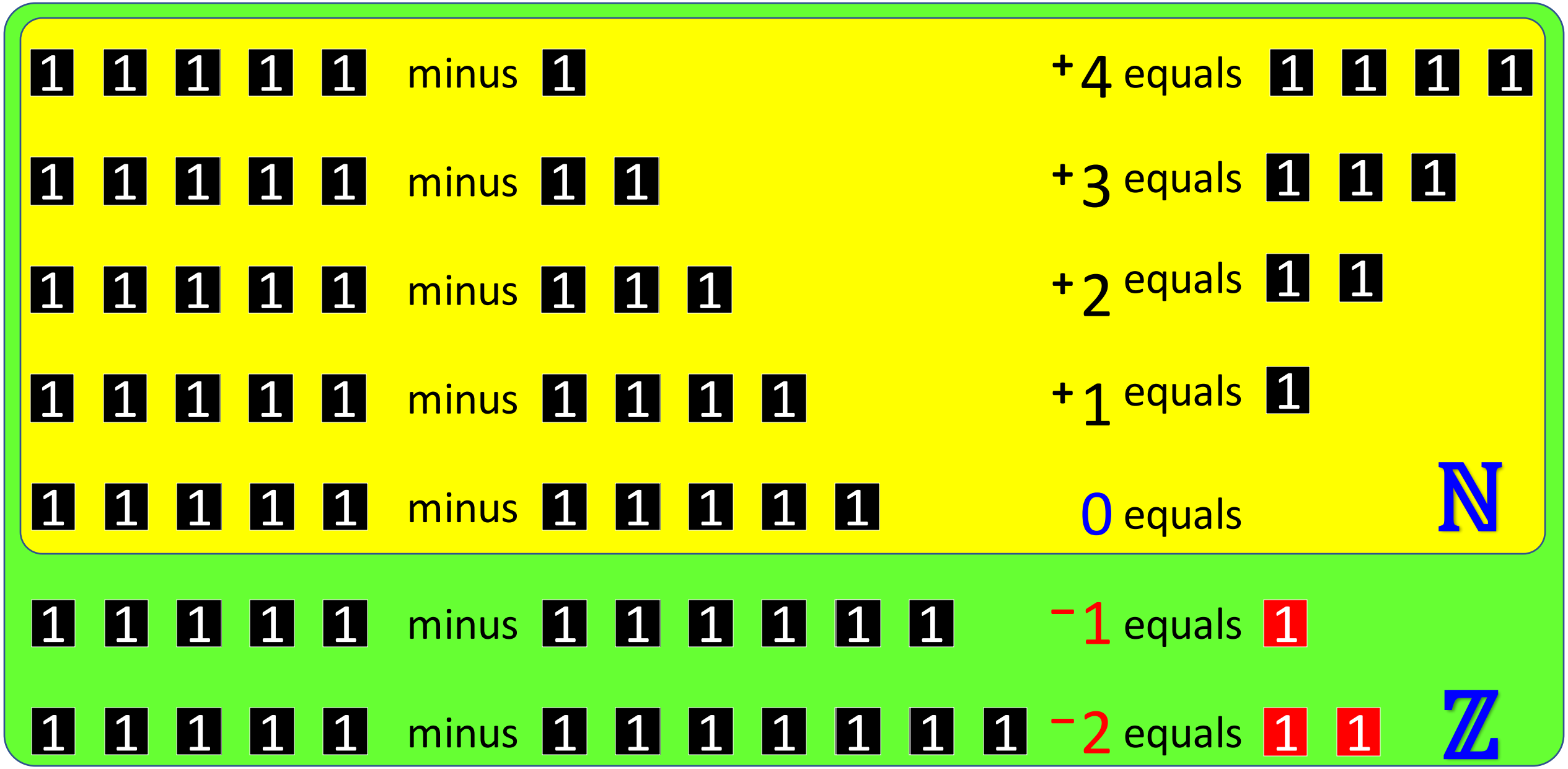
Negative Integers are less than Positive Integers.



The more left you go, the smaller the number. **NO!**

Maths pedagogies were developed on Naturals ℕ then extended via subtraction to form Integers ℤ

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

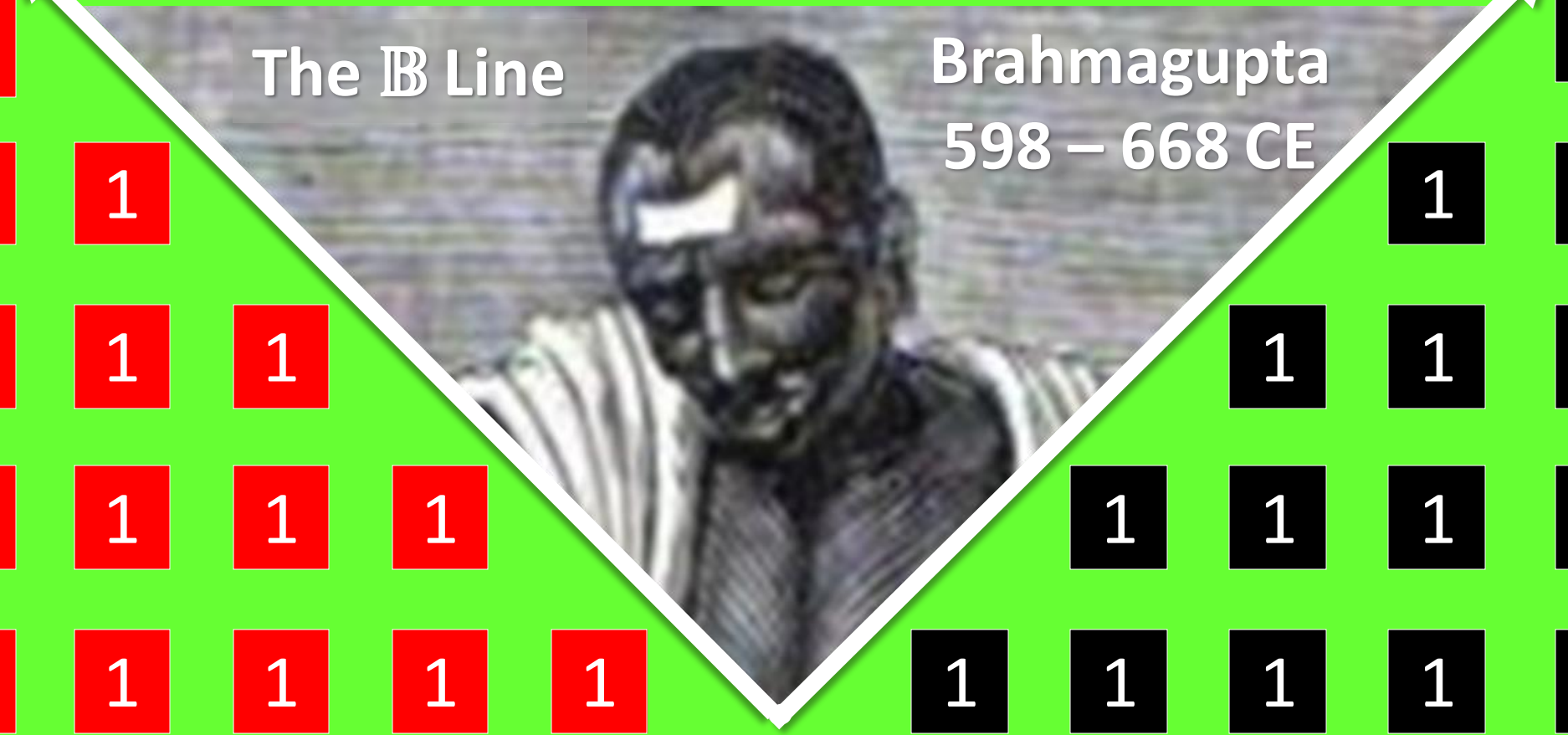


The West wrongly extended the Naturals pattern onto the Integers.

4 > 3 | 3 > 2 | 2 > 1 | 1 > 0 | 0 > ⁻1 | ⁻1 > ⁻2

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

“when *the number of* positive and negative *quantities* are equal the sum is zero”



5 4 3 2 1 0 1 2 3 4 5

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

© 2019 Jonathan J. Crabtree

Seeing Simple Symmetries of Quantity

Counts or measures of negative units

Counts or measures of positive units

Electrons South West Left Down Debts Loss
Deaths Emigration Cold Decay Below Zero
Less Than Enough Below Ground To the hour
Deceleration Head Wind (knots) Under Par (golf)

Positrons North East Right Up Assets Profit
Births Immigration Heat Growth Above Zero
More Than Enough Above Ground Past the hour
Acceleration Tail Wind (knots) Over Par (golf)

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

Brahmagupta's 5 Subtraction Laws

SL1 A smaller **positive** subtracted from a larger **positive** is positive.

$^{+}9 - {}^{+}2 = {}^{+}7$

SL2 A smaller **negative** subtracted from a larger **negative** is **negative**.

SL3 If a larger **negative** or **positive** is to be subtracted from a smaller **negative** or **positive**, the sign of their difference is reversed – **negative** becomes **positive** and **positive** **negative**.

$^{+}4 - {}^{+}6 = {}^{-}2$

SL4 A **negative** minus **zero** is **negative**,
a **positive** minus **zero** is **positive**,
zero minus **zero** is **zero**.

SL5 When a **positive** is to be subtracted from a **negative** or a **negative** from a **positive**, then it is to be added.

© 2019 J. J. Crabtree | www.jonathancrabtree.com

Brahmagupta's 5 Subtraction Laws

SL1 A smaller **positive** subtracted from a larger **positive** is **positive**.

SL2 A smaller **negative** subtracted from a larger **negative** is **positive**.

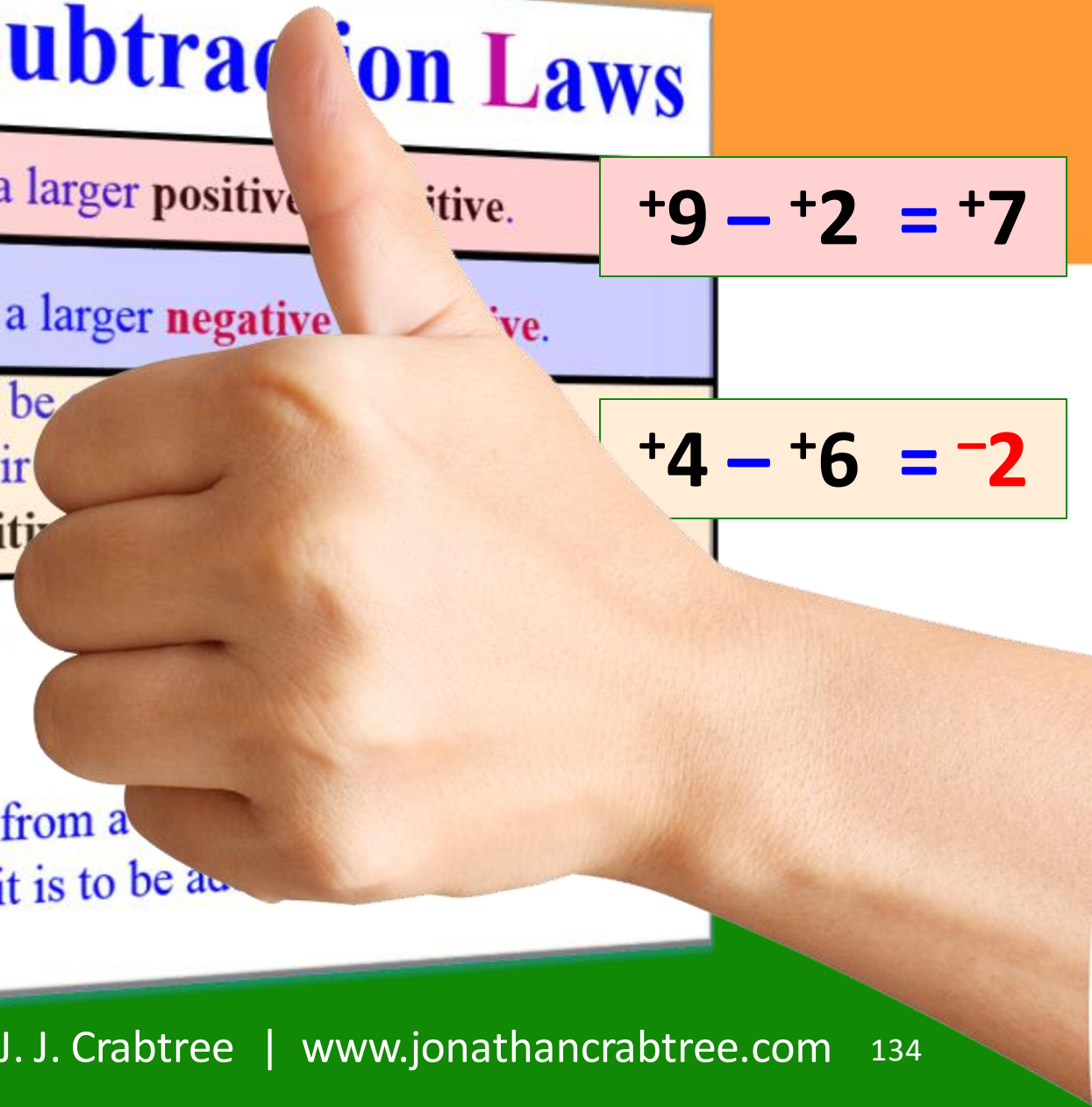
SL3 If a larger **negative** or **positive** is to be subtracted from a smaller **negative** or **positive**, the sign of their difference is **negative** becomes **positive** and positive becomes **negative**.

SL4 A **negative** minus **zero** is **negative**, a **positive** minus **zero** is **positive**, **zero** minus **zero** is **zero**.

SL5 When a **positive** is to be subtracted from a **negative** or a **negative** from a **positive**, then it is to be added.

$^{+}9 - {}^{+}2 = {}^{+}7$

$^{+}4 - {}^{+}6 = {}^{-}2$



© 2019 J. J. Crabtree | www.jonathancrabtree.com

Brahmagupta's 5 Subtraction Laws

SL1 A smaller **positive** subtracted from a larger **positive** is **positive**.

SL2 A smaller **negative** subtracted from a larger **negative** is **negative**.

SL3 If a larger **negative** or **positive** is to be subtracted from a smaller **negative** or **positive**, the sign of their difference is reversed – **negative** becomes **positive** and **positive** **negative**.

SL4 A **negative** minus **zero** is **negative**,
a **positive** minus **zero** is **positive**,
zero minus **zero** is **zero**.

SL5 When a **positive** is to be subtracted from a **negative** or a **negative** from a **positive**, then it is to be added.

Smaller Negative!

$^{-}8 - (^{-}5) = ^{-}3$

© 2019 J. J. Crabtree | www.jonathancrabtree.com

Brahmagupta's 5 Subtraction Laws

SL1 A smaller **positive** subtracted from a larger **positive** is **positive**.

SL2 A smaller **negative** subtracted from a larger **negative** is **negative**.

SL3 If a larger **negative** or **positive** is to be subtracted from a smaller **negative** or **positive**, the sign of their difference is reversed – **negative** becomes **positive** and **positive** **negative**.

SL4 A **negative** minus **zero** is **negative**,
a **positive** minus **zero** is **positive**,
zero minus **zero** is **zero**.

SL5 When a **positive** is to be subtracted from a **negative** or a **negative** from a **positive**, then it is to be added.

Smaller Negative!

$^{-}8 - (^{-}5) = {}^{-}3$

$^{-}3 - (^{-}7) = {}^{+}4$

Larger Negative!

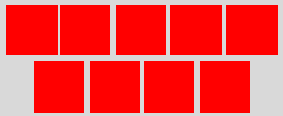
© 2019 J. J. Crabtree | www.jonathancrabtree.com

Brahmagu
n Laws
SL1
A smaller
SL2
A small
SL3
If a lar
negati
negati
SL4
A negat
a positiv
zero
SL5
When a posi
or a negative h
Smaller Negative!
$^{-}8 - {}^{-}5 = {}^{-}3$
$^{-}3 - {}^{-}7 = {}^{+}4$
Larger Negative!

Seeing Maths with Indian Eyes

Which numbers are **greater**?

or

 or

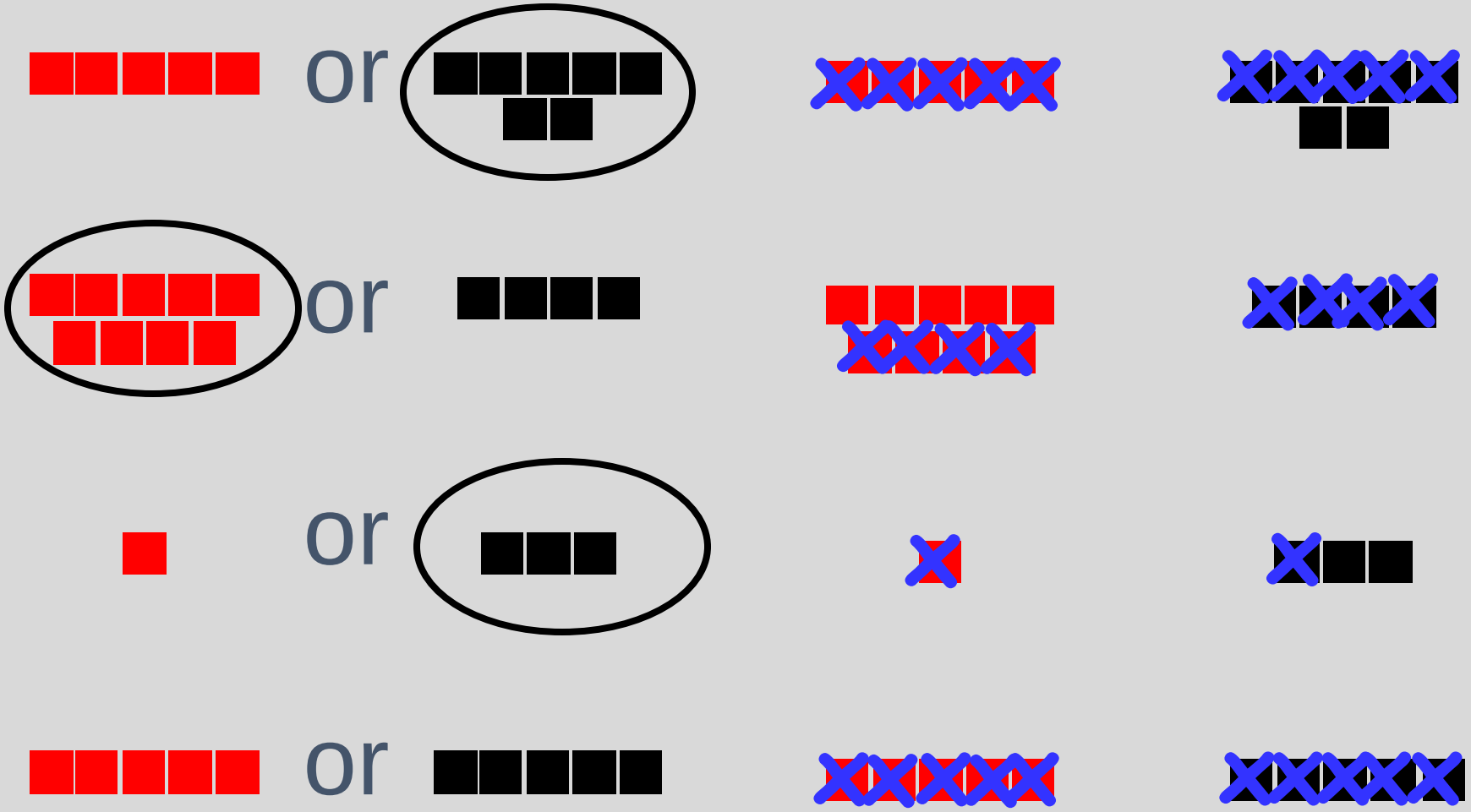
or

or

© 2019 Jonathan J. Crabtree

Seeing Maths with Indian Eyes

Which **numbers** are **greater**?

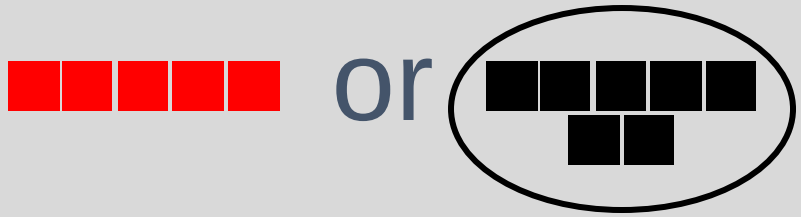


© 2019 Jonathan J. Crabtree

Seeing Maths with Indian Eyes

Which **numbers** are **greater**?

© 2019 Jonathan J. Crabtree

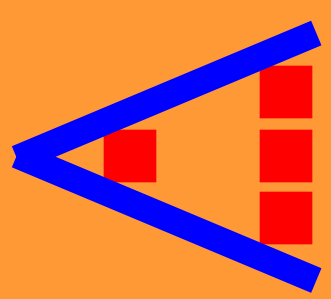


$5^- < 7^+$

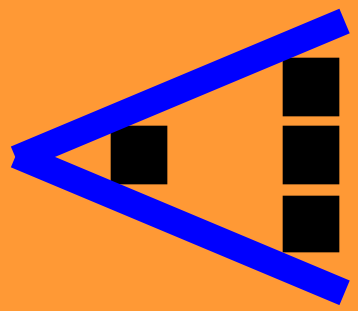
$9^- > 4^+$

$1^- < 3^+$

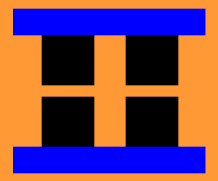
$5^- \not< 5^+$



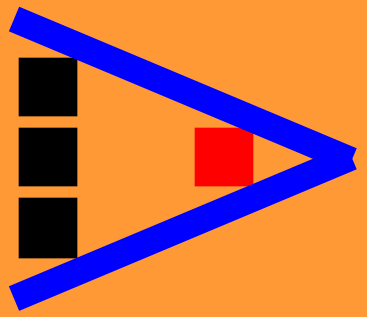
$1^- < 3^-$



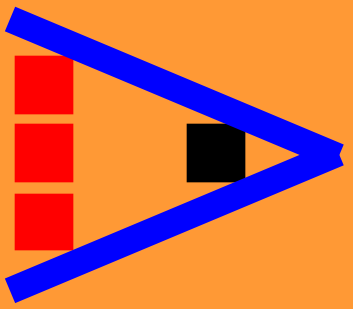
$1^+ < 3^+$



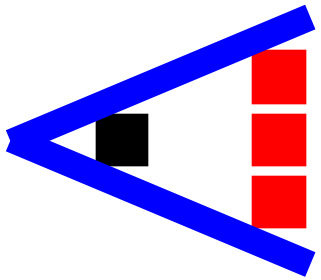
$2^+ = 2^+$



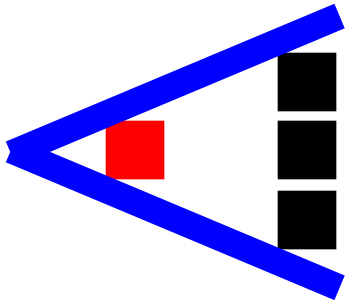
$3^+ > 1^-$



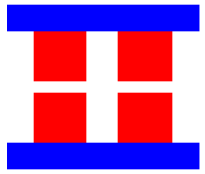
$3^- > 1^+$



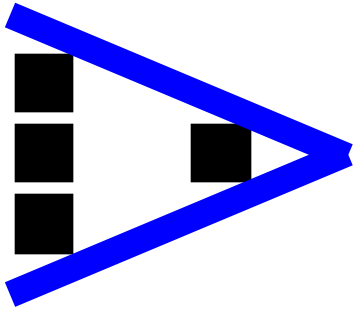
$1^+ < 3^-$



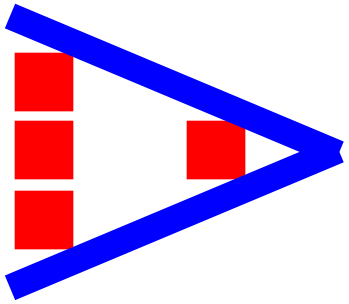
$1^- < 3^+$



$2^- = 2^-$



$3^+ > 1^+$



$3^- > 1^-$

2^- are equal & opposite to 2^+

$2^- \ominus 2^+$

© 2019 Jonathan J. Crabtree

JJC ASSESSMENT OF THE WORLD'S PEDAGOGICAL EVOLUTION (628 to Now)

$^{+}12 + {}^{+}4$	$^{+}12 + {}^{-}4$	$^{-}12 + {}^{+}4$	$^{-}12 + {}^{-}4$
$^{+}12 - {}^{+}4$	$^{+}12 - {}^{-}4$	$^{-}12 - {}^{+}4$	$^{-}12 - {}^{-}4$
$^{+}12 \times {}^{+}4$	$^{+}12 \times {}^{-}4$	$^{-}12 \times {}^{+}4$	$^{-}12 \times {}^{-}4$
$^{+}12 \div {}^{+}4$	$^{+}12 \div {}^{-}4$	$^{-}12 \div {}^{+}4$	$^{-}12 \div {}^{-}4$
→	PASS	FAIL	Absent

© 2019 J. J. Crabtree | www.jonathancrabtree.com

Podometic™ set to replace Arithmetic Dec. 2020

$^{+}12 + {}^{+}4$	$^{+}12 + {}^{-}4$	$^{-}12 + {}^{+}4$	$^{-}12 + {}^{-}4$
$^{+}12 - {}^{+}4$	$^{+}12 - {}^{-}4$	$^{-}12 - {}^{+}4$	$^{-}12 - {}^{-}4$
$^{+}12 \times {}^{+}4$	$^{+}12 \times {}^{-}4$	$^{-}12 \times {}^{+}4$	$^{-}12 \times {}^{-}4$
$^{+}12 \div {}^{+}4$	$^{+}12 \div {}^{-}4$	$^{-}12 \div {}^{+}4$	$^{-}12 \div {}^{-}4$
→	PASS	FAIL	Absent

© 2019 J. J. Crabtree | www.jonathancrabtree.com

Podometic™ set to replace Arithmetic Dec. 2020 with Free maths eBooks for every Indian child

$^{+}12 + {}^{+}4$	$^{+}12 + {}^{-}4$	$^{-}12 + {}^{+}4$	$^{-}12 + {}^{-}4$
$^{+}12 - {}^{+}4$	$^{+}12 - {}^{-}4$	$^{-}12 - {}^{+}4$	$^{-}12 - {}^{-}4$
$^{+}12 \times {}^{+}4$	$^{+}12 \times {}^{-}4$	$^{-}12 \times {}^{+}4$	$^{-}12 \times {}^{-}4$
$^{+}12 \div {}^{+}4$	$^{+}12 \div {}^{-}4$	$^{-}12 \div {}^{+}4$	$^{-}12 \div {}^{-}4$

PASS

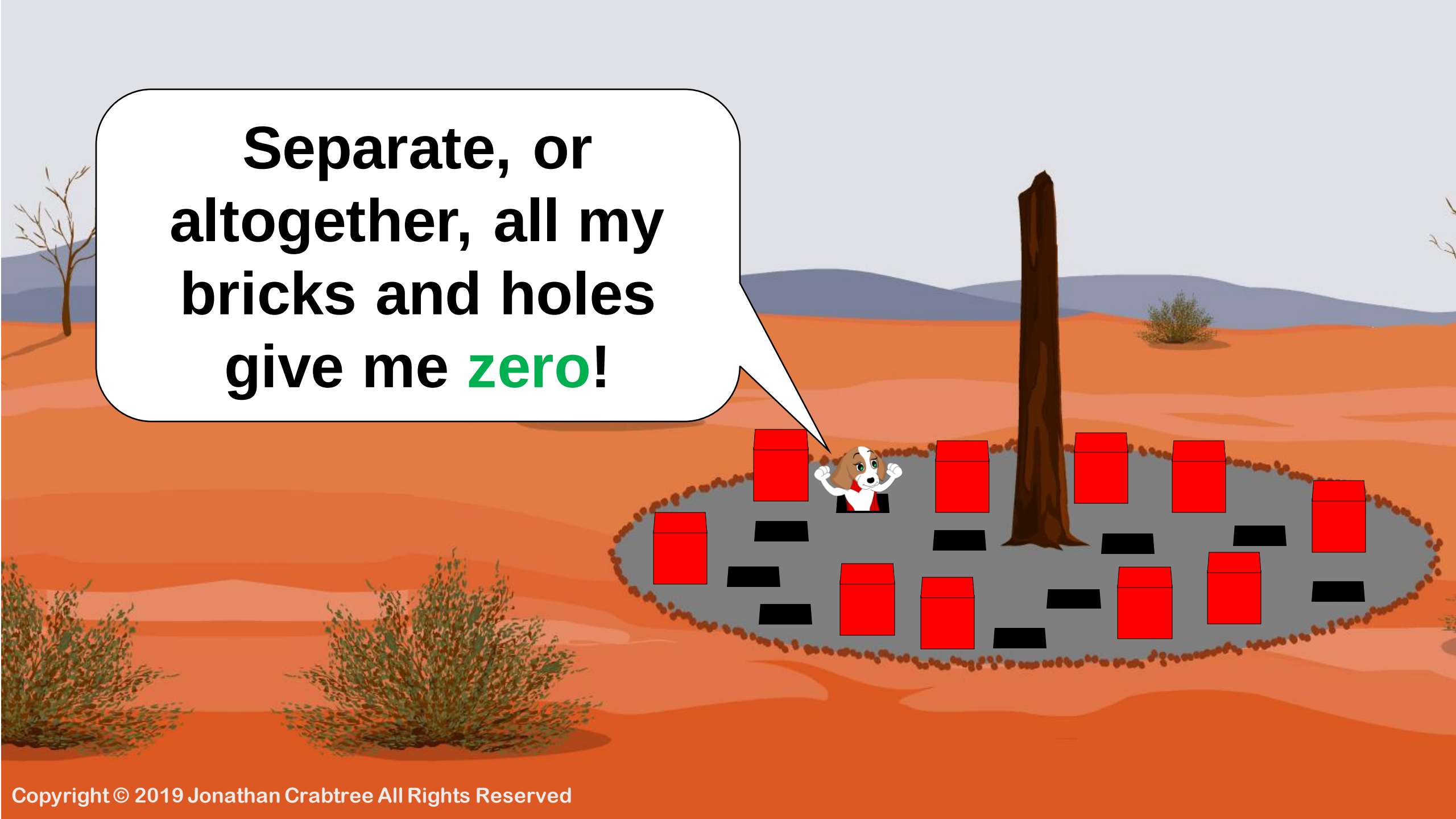
© 2019 J. J. Crabtree | www.jonathancrabtree.com

Copyright © 2019 Jonathan J. Crabtree All Rights Reserved

Hey kids, play with me and I'll make maths fun fast and free to learn!

From Class 1 on, it all connects with the laws of physics as well!

Copyright © 2019 Jonathan J. Crabtree All Rights Reserved



Copyright © 2019 Jonathan Crabtree All Rights Reserved

I'm Podo the Puppy.
All my maths adventures are being brought to life by AFX Animation in Kolkata.
www.afxanimation.com

Copyright © 2019 Jonathan J. Crabtree All Rights Reserved

The hard work has been done!

India can update its maths and prosper

or let this major maths education advantage pass by.

KEEP
DELHI
CLEAN

Big Problems Demand We

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

Think bigger

Sign the Petition!

www.j.mp/TeachIndianMaths

Local Legends

ANDREW MATHIESON

Brain power: Jonathan Crabtree has endless ideas for making the world a better place.
Picture: Tommy Ritchie 40442

BURIED among the hundreds of everyday emails in Jonathan Crabtree's inbox are a few worth keeping.

Replies from Nelson Mandela, Bishop Desmond Tutu, the Dalai Lama, even Muhammad Ali are quickly printed out and put aside for safe keeping.

They are a reminder of the power behind the written word; that ordinary people can make a difference.

"I haven't got one from the President of the United States or the Pope yet," Jonathan says, "but I'll get there soon.

"The emails are not really that important – it's much more about the ideas."

truck trying to beat a red light 25 years ago nearly left him dead on the road after colliding with his motorcycle.

Lying motionless on the ground, Jonathan could only remember the horizon spinning around as if he were in a plane going down.

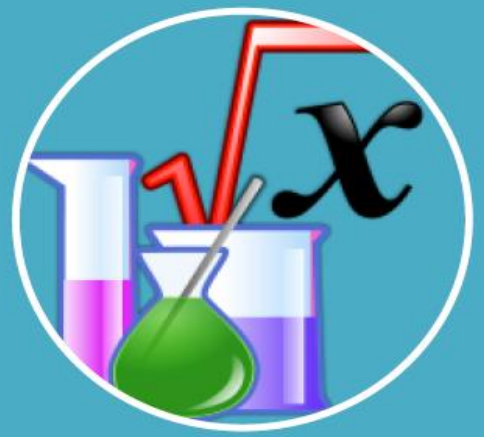
Consequently, the driver lost his licence but the

despite no formal qualifications, took up teaching mathematics from home.

"I actually failed maths," he laughs.

Throwing away the classroom text books, Jonathan taught kids to imagine algebra by closing their eyes and listening to fairytales.

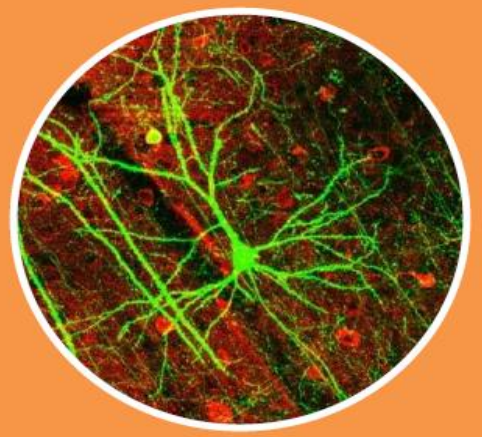
Education authorities in Australia first shunned



Science

Technology

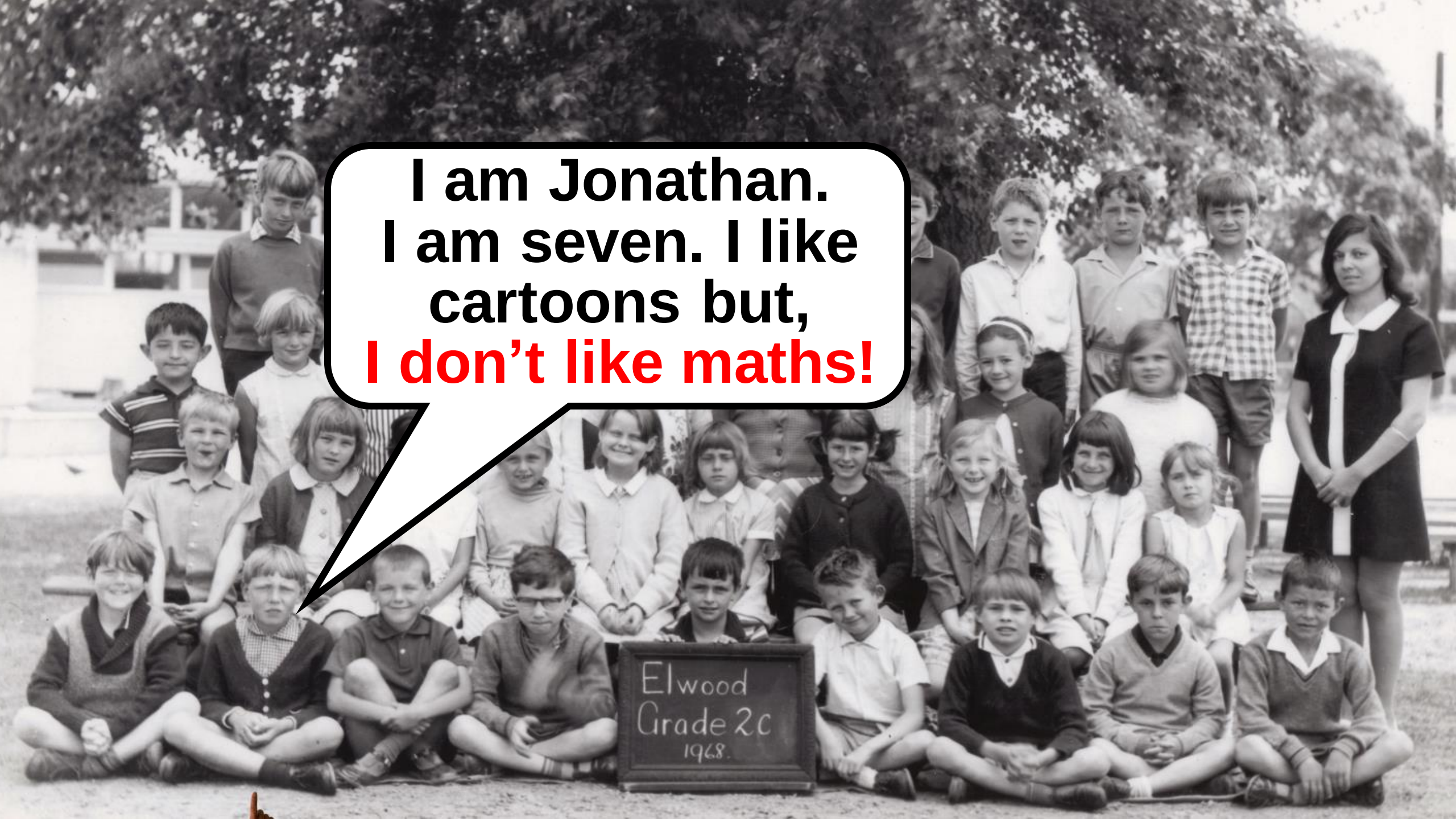
Engineering

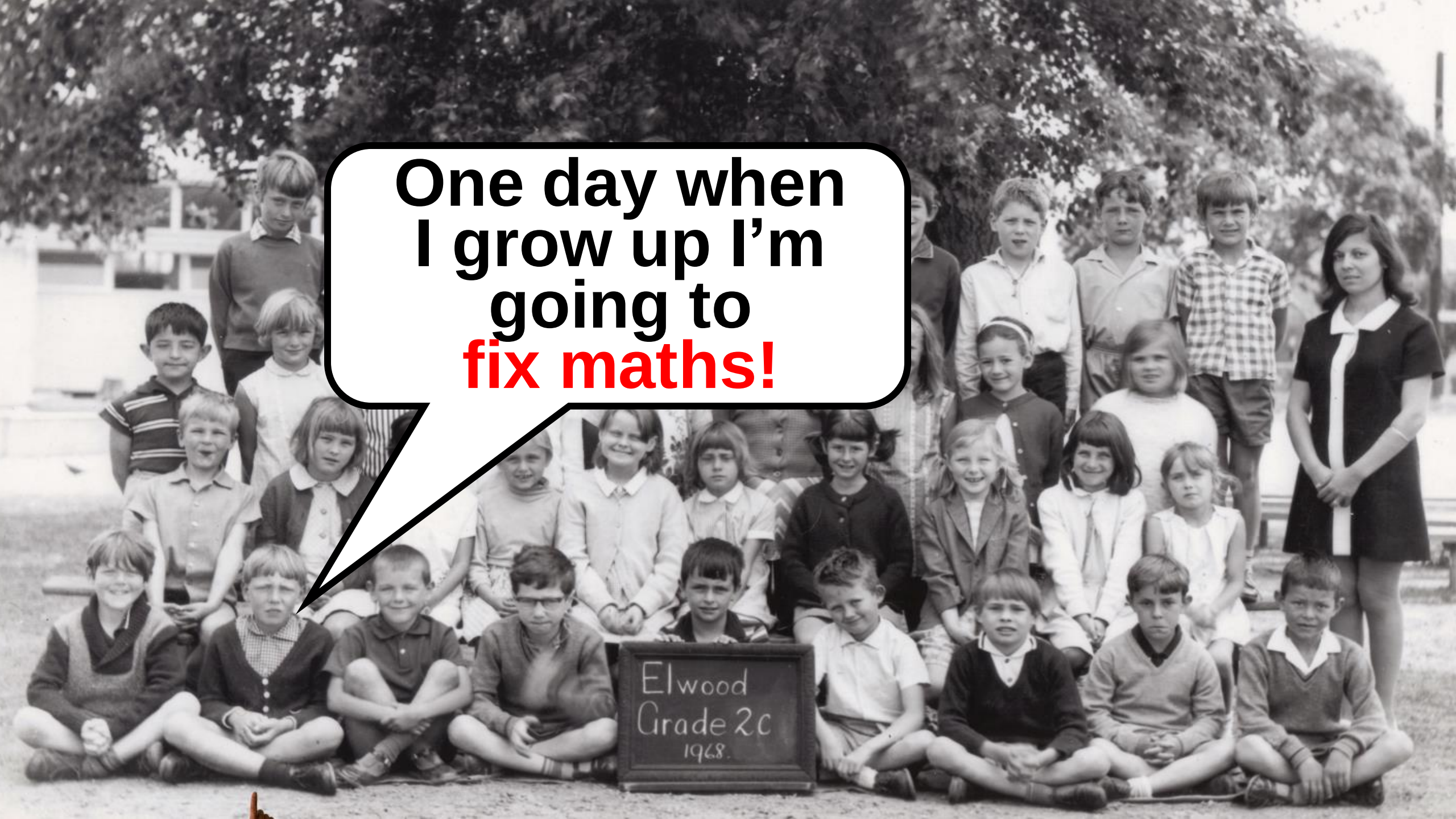


Medicine

In the West

MATHEMATICS


I am Jonathan.
I am seven. I like
cartoons but,
I don't like maths!
Elwood
Grade 2C
1968.


One day when I grow up I'm going to fix maths!
Elwood
Grade 2C
1968.

PISA Mathematics Survey?

(Programme for International Student Assessment)

In Tamil Nadu and Himachal Pradesh 15% and 12% of students are ready to use mathematics in ways that are considered fundamental for their future development.

https://www.acer.org/au/about-us/media/media-releases/acer-releases-results-of-pisa-2009-participant-economies

© 2019 J. J. Crabtree | www.jonathancrabtree.com

PISA Mathematics Survey?

(Programme for International Student Assessment)

In Tamil Nadu and Himachal Pradesh 15% and 12% of students are ready to use mathematics in ways that are considered fundamental for their future development.

The OECD average is 75%.

https://www.acer.org/au/about-us/media/media-releases/acer-releases-results-of-pisa-2009-participant-economies

© 2019 J. J. Crabtree | www.jonathancrabtree.com

PISA Science Survey?

(Programme for International Student Assessment)

In Tamil Nadu and Himachal Pradesh 16% and 11% of students are proficient in science … to participate actively in life situations related to science and technology.

https://www.acer.org/au/about-us/media/media-releases/acer-releases-results-of-pisa-2009-participant-economies

PISA Science Survey?

(Programme for International Student Assessment)

In Tamil Nadu and Himachal Pradesh 16% and 11% of students are proficient in science … to participate actively in life situations related to science and technology.

The OECD average is 82%.

https://www.acer.org/au/about-us/media/media-releases/acer-releases-results-of-pisa-2009-participant-economies

© 2019 J. J. Crabtree | www.jonathancrabtree.com

The English support the English cricket team

The English support the English cricket team

Indians support the Indian cricket team

© 2019 J. J. Crabtree | www.jonathancrabtree.com

The English support the English cricket team

Indians support the Indian cricket team

The English support English mathematicians

© 2019 J. J. Crabtree | www.jonathancrabtree.com

The English support the English cricket team

Indians support the Indian cricket team

The English support English mathematicians

Indians support English mathematicians!

Sign the Petition!

www.j.mp/TeachIndianMaths

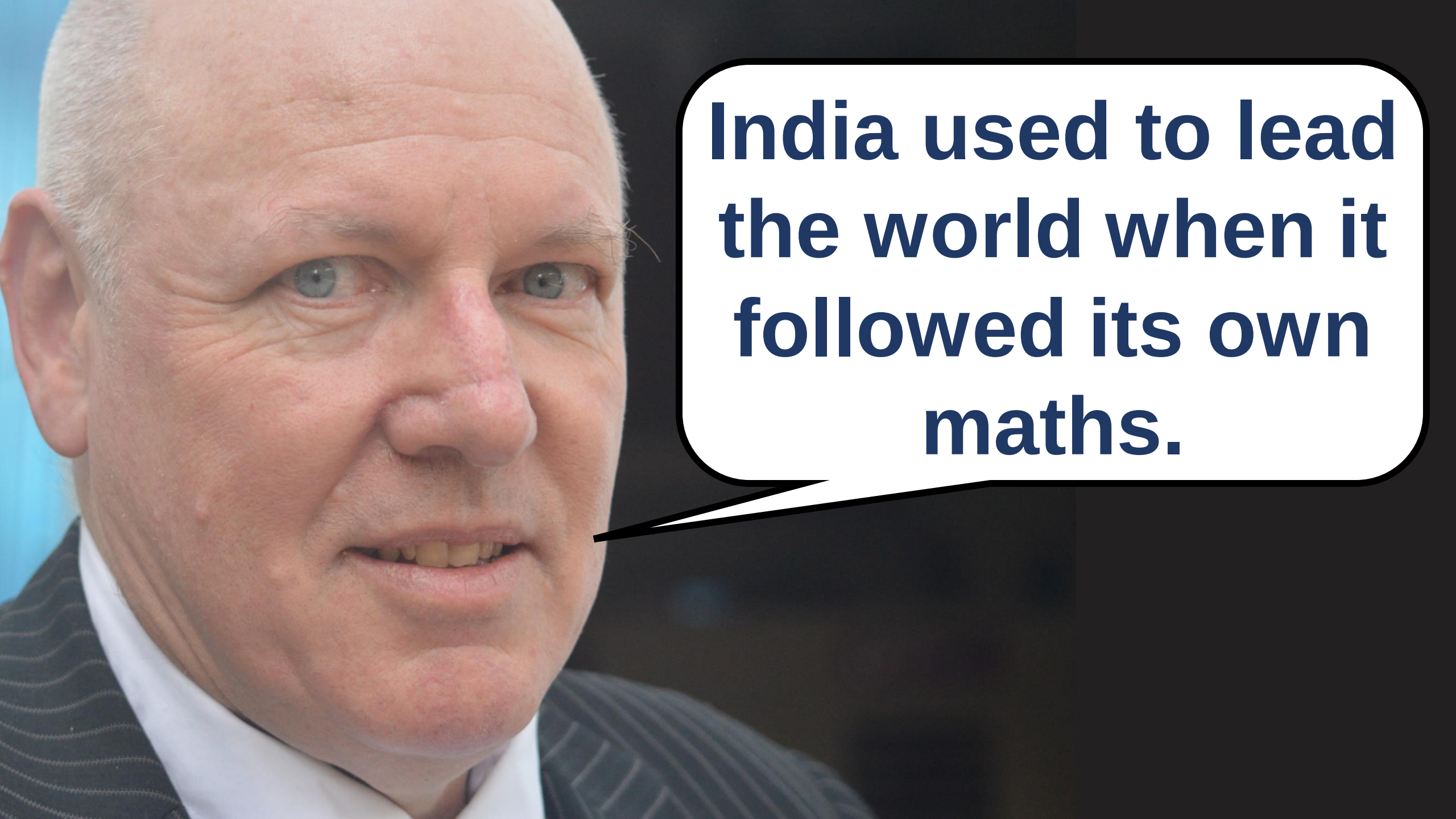
© 2019 J. J. Crabtree | www.jonathancrabtree.com

Mathematical Foundations?

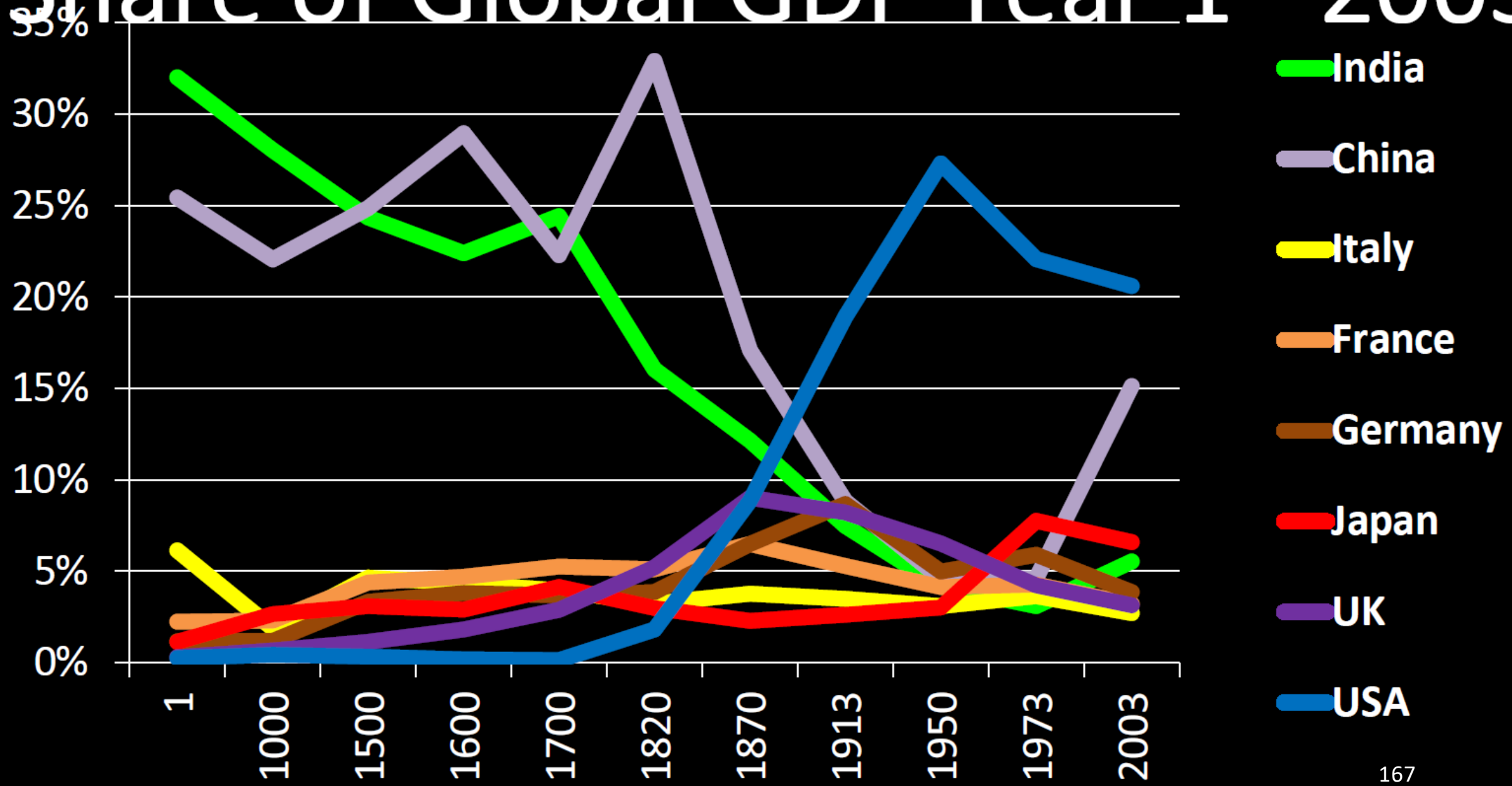
© 2019 J. J. Crabtree www.jonathancrabtree.com

TRUE!

FALSE!


India used to lead the world when it followed its own maths.

Share of Global GDP Year 1 - 2003



The hard work has been done!

India can update its maths and prosper

or let this major maths education advantage pass by.

Thank you India for your gift of zero. It got lost, so I have returned it to you.

If you care,
please share!
Thank you.

PART 2. Negative & Positive Quantities on a Brahmaguptan Plane for India's Primary Classes

Jonathan J. Crabtree, Elementary Mathematics Historian, Australia.

9th National conference: Technology & Innovations in Math Education
Biennial Conference of the Mathematics Department of the IIT Bombay.
Jointly organized by IISER, Pune & BATU, Lonere.
Pune India, 27th December 2019

“There are no studies demonstrating how negative numbers and algebra can be taught to such students in a meaningful way.”

“There are no studies demonstrating how negative numbers and algebra can be taught to such students in a meaningful way.”

Mathematics Education in India *Status and Outlook*

Edited by R. Ramanujam and K. Subramaniam

Homi Bhabha Centre for Science Education

Tata Institute of Fundamental Research

“Negative numbers, usually introduced early in class 6, are known to be a problem area.”

Mathematics Education in India *Status and Outlook*

Edited by R. Ramanujam and K. Subramaniam

Homi Bhabha Centre for Science Education

Tata Institute of Fundamental Research

“Negative times negative is positive is problematic to justify.”

A quote I jotted down from Dr. Shailesh Shirali this morning, 27 December.

“Negative times negative is positive is problematic to justify.”

Now, Brahmagupta’s simple idea of zero lets India’s children understand $^{-}1 \times -1 = {}^{+}1$ with absolute rock-solid certainty and intuitive conviction!

“Something is amiss. Why are we adrift?
What should be done?
We need to gain insight from history.”

Some quotes I jotted down from Professor Dinesh Singh’s inaugural TIME2019 address 26 December.

“We must offer something tangible to policy holders. Sanskrit is in our DNA, but we never bring it into the learning of mathematics”

Some quotes I jotted down from Professor Dinesh Singh’s inaugural TIME2019 address 26 December.

“We must offer something tangible to policy holders. Sanskrit is in our DNA, but we never bring it into the learning of mathematics”

My goal is to answer questions like these and to give India’s leaders and teachers the solutions children need.

“We must offer something tangible to policy holders. Sanskrit is in our DNA, but we never bring it into the learning of mathematics”

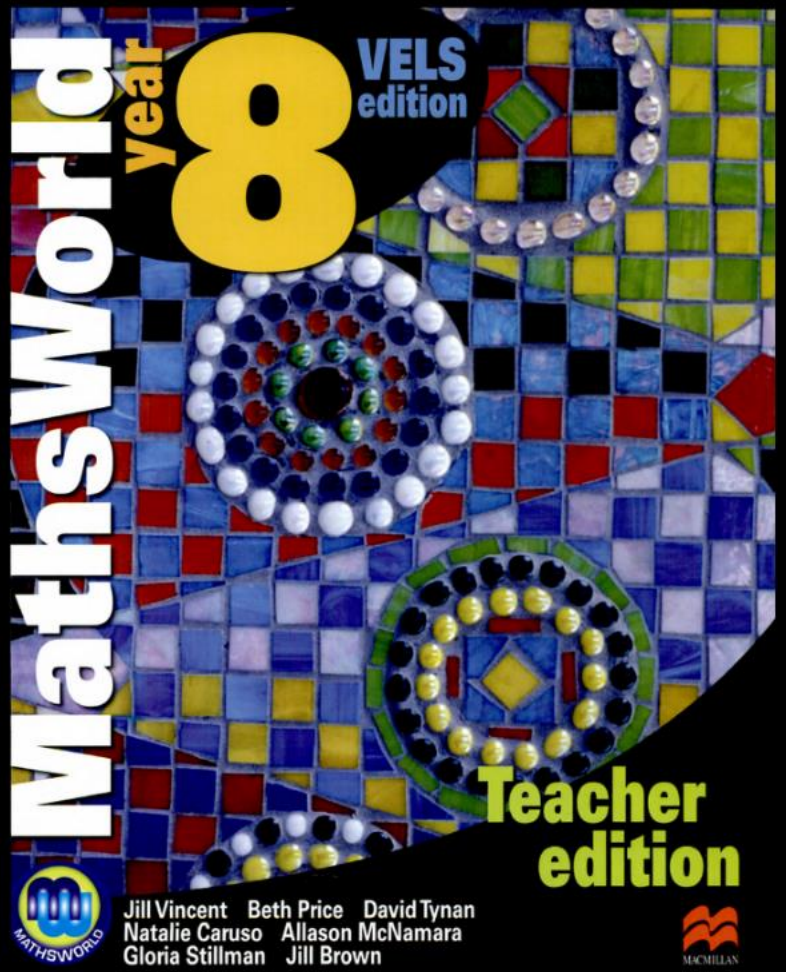
To solve India’s primary-level mathematics education problems, I am creating a free ‘Sanskrit-based’ alternative to Arithmetic called Podometic™.

“Please forget everything you learned in school, because you have not learned it.”

Edmund Landau, *Foundations of Analysis*

Please forget much about what you learned in school about arithmetic, because you don't understand my definition of zero.

What Brahmagupta might say if he were alive today.

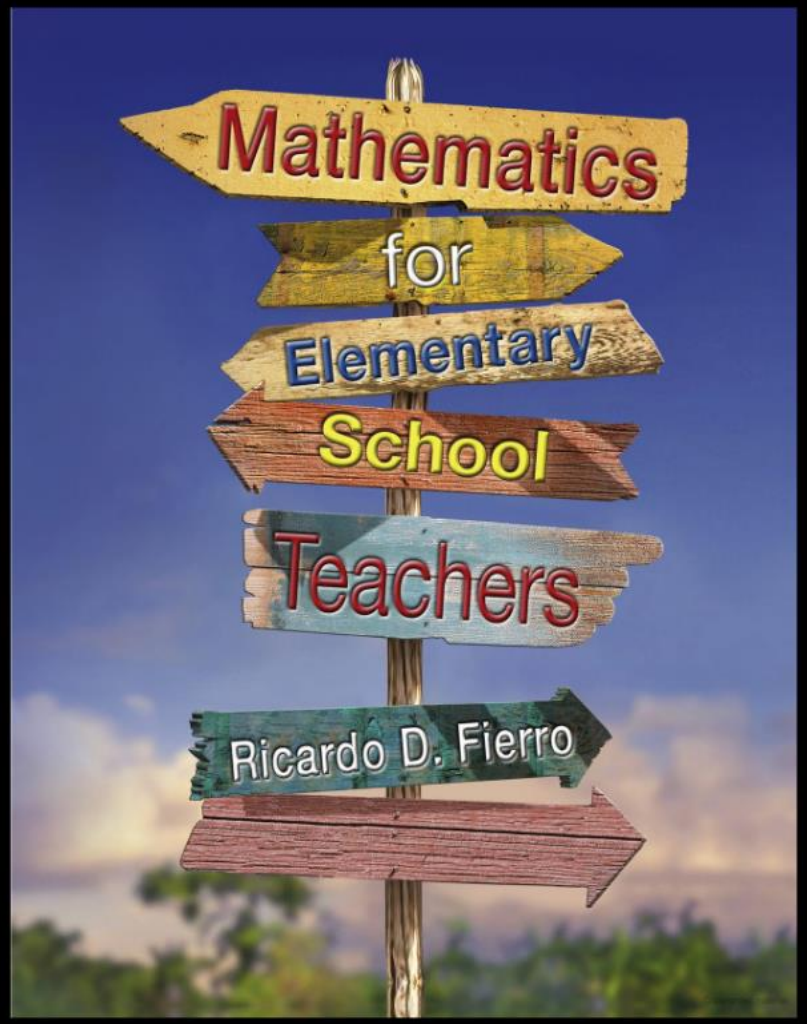


"Brahmagupta... defined zero as the result of subtraction of a number from itself."

"He also gave the following rules for operations on what he called 'fortunes' (positive numbers) and 'debts' (negative numbers)."

"The product... of two debts is one fortune."

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com



"Brahmagupta (598 – 670 CE) was a Hindu mathematician and astronomer who lived in the first century."

"He used negative integers to represent debts and positive integers to represent assets."

"The product ... of two debts is one fortune."

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

Brahmagupta

"The product of a positive and a negative (number) is negative; of two negatives is positive; positive multiplied by 'positive is positive.“

Mahāvīra

"In the multiplication of two negative or two positive numbers the result is positive; but it is negative in the case of (the multiplication of) a positive and a negative number."

Śrīpati

"On multiplying two negative or two positive numbers (the product is) positive; in the multiplication of positive and negative (the result is) negative."

Bhāskara II

"The product of two positive or two negative (numbers) is positive; the product of positive and negative is negative.” The same rule is stated by Nārāyaṇa.

SOURCE: *History of Hindu Mathematics: A Source Book, Part II, Algebra*, Bibhutibhusan Datta and Bidyāraṇya Avadesh Narayan Singh, pp. 22-23, Motilal Banarsidass, Lahore, 1938.

In 263 CE, Liu Hui wrote a commentary on the ancient
The Nine Chapters on the Mathematical Art
(九章算术 Jiǔzhāng Suànshù circa 100 CE).

In 263 CE, Liu Hui wrote a commentary on the ancient ***The Nine Chapters on the Mathematical Art*** (九章算术 Jiǔzhāng Suànshù circa 100 CE).

Liu Hui said:
I read the Nine Chapters as a boy, and studied it in full detail when I was older. I observed the division between the dual natures of Yin and Yang [the negative and positive aspects] which sum up the fundamentals of mathematics.

In 263 CE, Liu Hui wrote a commentary on the ancient ***The Nine Chapters on the Mathematical Art*** (九章算术 Jiǔzhāng Suànshù circa 100 CE).

Liu Hui said:
I read the Nine Chapters as a boy, and studied it in full detail when I was older. I observed the division between the dual natures of Yin and Yang [the negative and positive aspects] which sum up the fundamentals of mathematics.

The Nine Chapters on the Mathematical Art: Companion and Commentary,
Shen Kangshen, John N. Crossley and Anthony W. C. Lun, Oxford University Press, 2000.

If Maths is Language,
it must obey the
‘Parts of Speech’
of language.

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

The Grammar of Arithmetic

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

Negative Seven Minus Negative Four?

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

Most adults answer *“Negative Eleven?”*

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

Most adults answer *"Negative Eleven?"*

Why?

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

Adjective Adjective

Negative Seven

Verb

Minus

Adjective Adjective

Negative Four?

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

Adjective Adjective

Negative Seven

Verb

Minus

Adjective Adjective

Negative Four?

No Nouns!

© 2015 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

Adjective Adjective

Negative Seven

Verb

Minus

Adjective Adjective

Negative Four?

Adults struggle for rules memorized without meaning *‘is it two negatives make a plus?’*

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

Seven Negatives Minus Four Negatives?

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

Every child answers

“Three Negatives!”

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

Every child answers

“Three Negatives!”

Why?

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

Adjective Nouns

Seven Negatives

Verb

Minus

Adjective Nouns

Four Negatives?

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

Adjective Nouns

Seven Negatives

Verb

Minus

Adjective Nouns

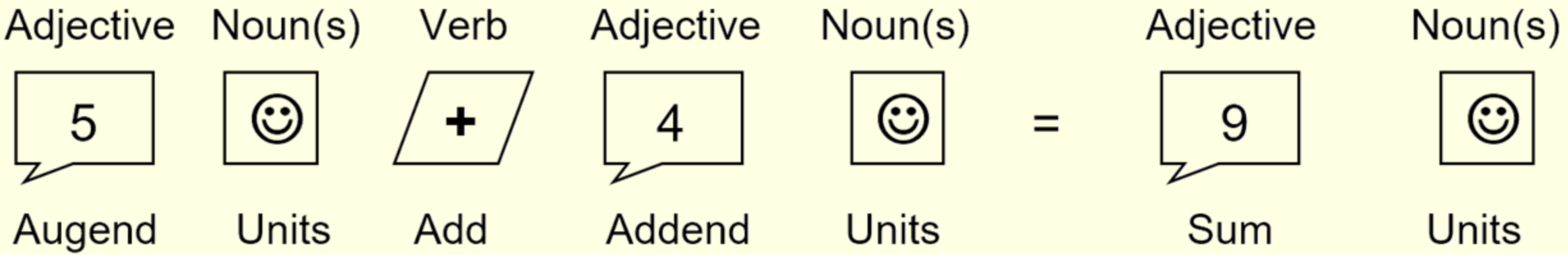
Four Negatives?

Nouns Make Maths Meaningful!

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

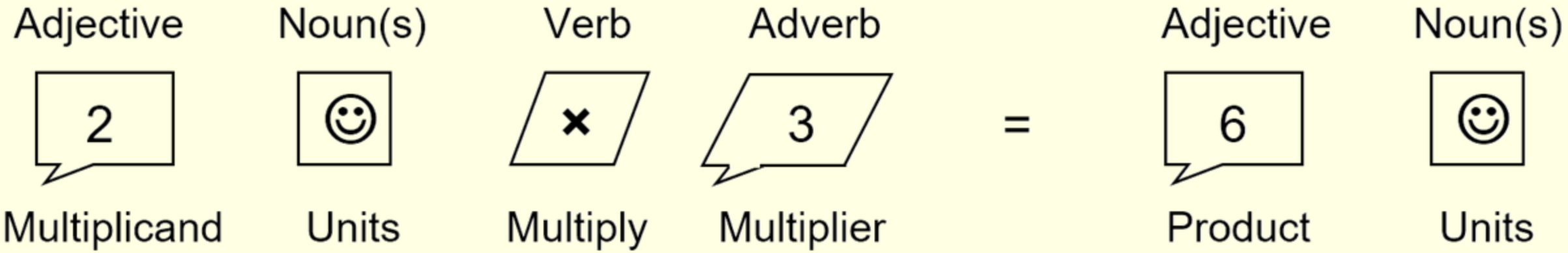
Addition



www.j.mp/IndiasMaths

© 2019 J. J. Crabtree | All Rights
Reserved www.podometic.in

Multiplication



www.j.mp/2020Maths

www.jonathancrabtree.com

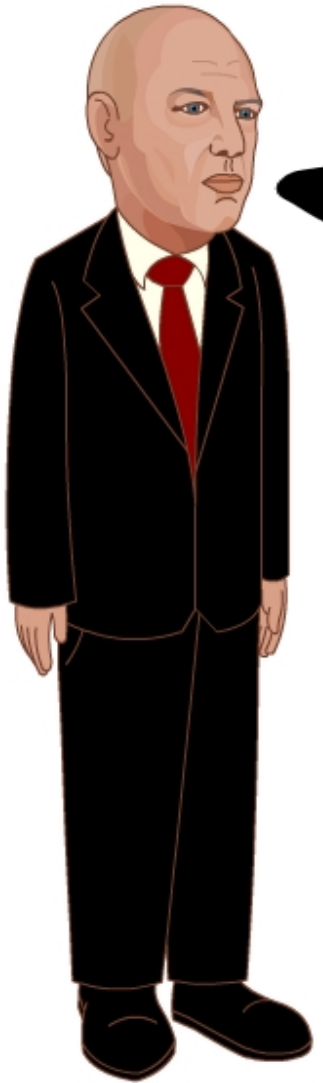
Why the product of two negatives is positive

A demonstration goes like this…

$1 + (-1) = 0$	Definition of -1.
$-1 \times [1 + (-1)] = -1 \times 0$	Both sides multiplied by -1.
$(-1) \times 1 + (-1) \times (-1) = 0$	Distributive law
$(-1) + (-1) \times (-1) = 0$	Multiplicative identity
$1 + [(-1) + (-1) \times (-1)] = 1 + 0$	Add both sides to 1.
$[1 + (-1)] + (-1) \times (-1) = 1 + 0$	Associative law
$0 + (-1) \times (-1) = 1 + 0$	Definition of -1
$(-1) \times (-1) = 1$	Additive identity

Crabtree, Jonathan J. *Snippet: A new reason negative multiplied by negative is positive* Vinculum, Vol. 52, No. 3, Jul 2015

© 2019 Jonathan J. Crabtree



Brahmagupta defined zero as the sum of a positive number and negative number of equal magnitude, सम-ऐक्यम् खम् (Brāhma Sphuta-siddhānta, Chapter 18:30a).

So zero was defined in India as:

$^{+}n + {}^{-}n$

Copyright © 2019 Jonathan J. Crabtree All Rights Reserved

P.1

अथ धनर्णशून्यानां सङ्कलनम् ।

धनयोर्धनमृणमृणयो-
र्धनर्णयोरन्तरं समैक्यं खम् ।
ऋणमैक्यं च धनमृणध-
नशून्ययोः शून्ययोः शून्यम् ॥ ३० ॥ (३१)

धनयोरैक्यं धनमृणयोरैक्यमृणं भवति । धनर्णयोरन्तरमेवैक्यं भवति । समयोर्धनर्णयोरैक्यं खं शून्यं भवति । ऋणशून्ययोरैक्यमृणं धनशून्ययोरैक्यं धनं शून्ययोरैक्यं च शून्यं भवति ।

अत्रोपपत्त्यर्थं मन्मुद्रिता भास्करबीजटिप्पणी द्रष्टव्या ॥ ३० ॥

इदानीं व्यवकलनमाह ।

ऊनमधिकाद्विशोध्यं धनं धनादृणमृणादधिकमूनात् ।
व्यस्तं तदन्तरं स्यादृणं धनं धनमृणं भवति ॥ ३१ ॥ (३२)
शून्यविहीनमृणमृणं धनं धनं भवति शून्यमाकाशम् ।
शोध्यं यदा धनमृणादृणं धनाद्वा तदा क्षेप्यम् ॥ ३२ ॥ (३३)

अधिकादूनादूनं धनं विशोध्यं शेषं धनं भवति । अधिकादृणादूनमृणं विशोध्यं शेषमृणं भवति । ऊनाद्धनादधिकं धनं वोनादृणादधिकमृणं विशोध्यं तदा तदन्तरं व्यस्तं विपरीतं स्यात् । अर्थादधिकं धनं विशोध्यं तदा शेषमृणं भवति । अधिकमृणं विशोध्यं तदा शेषं धनं भवति । कथं विपरीतं भवतीत्याह । ऋणं धनं भवति धनं चर्णं भवतीति । चेदृणं शून्यविहीनं शून्येन विहीनं तदा ऋणं धनं च शून्यविहीनं धनं शून्यं च शून्यविहीनमाकाशं शून्यं भवति । यदि ऋणादूनं शोध्यं वा धनादृणं शोध्यं तदा क्षेप्यमर्थात् तदा तयोर्योग एवान्तरं भवतीति ।

अत्रोपपत्त्यर्थं मन्मुद्रिता भास्करबीजटिप्पणी विलोक्या ॥ ३१–३२ ॥

© 2019 J. J. Crabtree | www.jonathancrabtree.com

P.2

इदानीं गुणने करणसूत्रम् ।

ऋणमृणधनयोर्घातो धनमृणयोर्धनवधो धनं भवति ।
शून्यर्णयोः स्वधनयोः स्वशून्ययोर्वा वधः शून्यम् ॥ ३३ ॥ (३४)

ऋणधनयोर्घात ऋणं भवति । ऋणयोर्वधो धनवधो धनयोर्वधश्च धनं भवति । शून्यर्णयोः स्वधनयोः शून्यधनयोर्वा स्वशून्ययोश्च वधः शून्यं भवति ॥ ३३ ॥

इदानीं भागहारे करणसूत्रं वृत्तद्वयम् ।

धनभक्तं धनमृणहृतमृणं धनं भवति स्वं स्वभक्तं स्वम् ।
भक्तमृणेन धनमृणं धनेन हृतमृणमृणं भवति ॥ ३४ ॥ (३५)
खोद्धृतमृणं धनं वा तच्छेदं स्वमृणधनविभक्तं वा ।
ऋणधनयोर्वर्गः स्वं स्वं स्वस्य पदं कृतिर्यत् तत् ॥ ३५ ॥ (३६)

धनं धनभक्तं वा ऋणं ऋणभक्तं फलं धनं भवति । स्वभक्तं स्वं फलं स्वं भवति । ऋणेन धनं भक्तं फलमृणं स्यात् । धनेन ऋणं हृतं फलमृणं भवति । ऋणं वा धनं खेनोद्धृतं तच्छेदं तस्य शून्यस्य छेदो यस्मिन्नृणे वा धने तच्छेदं भवति । एवं स्वं शून्यमृणधनविभक्तं (शून्यं) वा तच्छेदं भवति । फलं शून्यं भवति वा शून्यं तद्धरं स्यादित्यर्थः । ऋणधनयोर्वर्गः स्वं भवति । स्वस्य वर्गः स्वं भवति । तदेव वर्गस्य पदं भवति यत्कृतिः स एव वर्गो भवेदिति । भास्करबीजेऽप्येतदेव सर्वम् । अत्र स्वभक्तं स्वमर्थात् $\frac{0}{0}$ इदं सर्वदा शून्यसमं नेत्येतदर्थं चलनकलनं विलोक्यम् ॥ ३४–३५ ॥

इदानीं सङ्क्रमणविषमकर्माह ।

योगोऽन्तरयुतहीनो द्विहृतः सङ्क्रमणमन्तरविभक्तं वा ।
वर्गान्तरमन्तरयुतहीनं द्विहृतं विषमकर्म ॥ ३६ ॥ (३७)

योगो राश्योर्योगोऽन्तरेण राश्यन्तरेण युतो हीनश्च द्विहृतो दलितो राशी स्तः । इदं सङ्क्रमणं नाम गणितम् । वा राश्योर्वर्गान्तरं राश्यन्तरेण विभक्तं फलमन्तरेण युतं हीनं द्विहृतं च राशी स्तः । इदं विष-

Brahmagupta's 5 Addition Laws

AL1 **positive** plus **positive** is **positive**

AL2 **negative** plus **negative** is **negative**

AL3 **positive** plus **negative** is the difference between the **positive** and **negative**

AL4 when **positive** and **negative** are equal the sum is **zero**

AL5
positive plus **zero** is **positive**
negative plus **zero** is **negative**
zero plus **zero** is **zero**

Brahmagupta's 5 Subtraction Laws

SL1 A smaller **positive** subtracted from a larger **positive** is **positive**.

SL2 A smaller **negative** subtracted from a larger **negative** is **negative**.

SL3 If a larger **negative** or **positive** is to be subtracted from a smaller **negative** or **positive**, the sign of their difference is reversed – **negative** becomes **positive** and **positive** **negative**.

SL4
A **negative** minus **zero** is **negative**,
a **positive** minus **zero** is **positive**,
zero minus **zero** is **zero**.

SL5 When a **positive** is to be subtracted from a **negative** or a **negative** from a **positive**, then it is to be added.

Brahmagupta's 4 Multiplication Laws

ML1 The product of a **negative** and a **positive** is **negative**.

ML2 The product of two **negatives** is **positive**.

ML3 The product of two **positives** is **positive**.

ML4
The product of **zero** and a **negative**,
of **zero** and a **positive**, or
of two **zeros** is **zero**.

© 2019 J. J. Crabtree | All Rights Reserved www.jonathancrabtree.com

Brahmagupta's 4 Division Laws

DL1 A **positive** divided by a **positive** is **positive**.

DL2 A **negative** divided by a **negative** is **positive**.

DL3 A **positive** divided by a **negative** is **negative**.

DL4 A **negative** divided by a **positive** is **negative**.

Acknowledgement: I am grateful to Avinash Sathaye, K. Ramasubramanian, Clemency Montelle, Kim Plofker and Agathe Keller. Analysis, interpretation *(& any mistakes)* by Jonathan J. Crabtree.

Seeing a Simple Proof for Brahmagupta's ML2

The product of two 'dashed numbers' is positive.

Via Brahmagupta we multiply **positives** and **negatives** by either adding to **zero** multiple times **or** subtracting from **zero** multiple times.

© 2019 Jonathan J. Crabtree

Seeing a Simple Proof for Brahmagupta's ML2

The product of two 'dashed numbers' is positive.

Via Brahmagupta we multiply **positives** and **negatives** by either adding to **zero** multiple times **or** subtracting from **zero** multiple times.

$^{-}a \times +b =$

^{-}a *added to*

zero b *times*

$^{-}a \times -b =$

^{-}a *subtracted from*

zero b *times*

© 2019 Jonathan J. Crabtree

Seeing a Simple Proof for Brahmagupta's ML2

The product of two 'dashed numbers' is positive.

Via Brahmagupta we multiply **positives** and **negatives** by either adding to **zero** multiple times **or** subtracting from **zero** multiple times.

$^{-}a \times +b =$

^{-}a *added to*

zero b *times*

$^{-}a \times -b =$

^{-}a *subtracted from*

zero b *times*

With integral multiplication…

a **is the *adjective* describing counts or measures of *noun* quantities**

b **is the *adverb* describing *verb* counts of additions or subtractions**

© 2019 Jonathan J. Crabtree

Seeing a Simple Proof for Brahmagupta's ML2

The product of two 'dashed numbers' is positive.

$^{-}a \times -b = {}^{-}a$ *subtracted from zero* b *times*

$^{-}1 \times -1 = {}^{-}1$ *subtracted from zero* 1 *time*

© 2019 Jonathan J. Crabtree

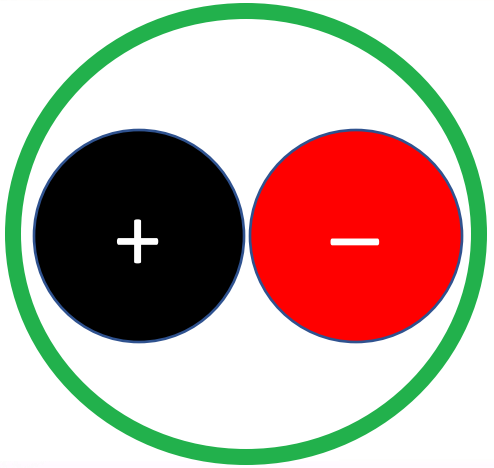
Seeing a Simple Proof for Brahmagupta's ML2

The product of two 'dashed numbers' is positive.

$^{-}a \times -b = {}^{-}a$ *subtracted from zero* b *times*

$^{-}1 \times -1 = {}^{-}1$ *subtracted from zero* 1 *time*

$^{+}1 + {}^{-}1$



$1 + (-1) = 0$	Definition of -1.
$-1 \times [1 + (-1)] = -1 \times 0$	Both sides multiplied by -1.
$(-1) \times 1 + (-1) \times (-1) = 0$	Distributive law
$(-1) + (-1) \times (-1) = 0$	Multiplicative identity
$1 + [(-1) + (-1) \times (-1)] = 1 + 0$	Add both sides to 1.
$[1 + (-1)] + (-1) \times (-1) = 1 + 0$	Associative law
$0 + (-1) \times (-1) = 1 + 0$	Definition of -1
$(-1) \times (-1) = 1$	Additive identity

© 2019 Jonathan J. Crabtree

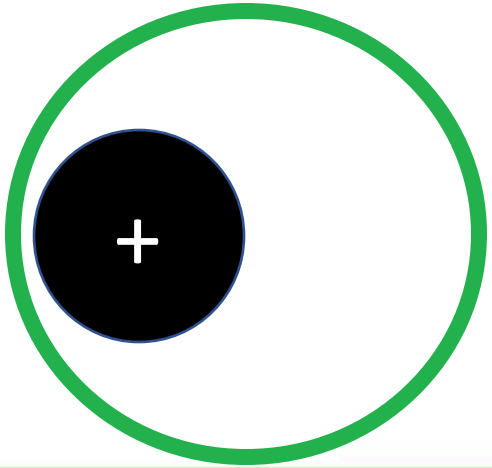
Seeing a Simple Proof for Brahmagupta's ML2

The product of two 'dashed numbers' is positive.

$^{-}a \times -b = {}^{-}a$ *subtracted from zero* b *times*

$^{-}1 \times -1 = {}^{-}1$ *subtracted from zero* 1 *time*

$$^{-}1 \times -1 = {}^{+}1$$



$1 + (-1) = 0$	Definition of -1.
$-1 \times [1 + (-1)] = -1 \times 0$	Both sides multiplied by -1.
$(-1) \times 1 + (-1) \times (-1) = 0$	Distributive law
$(-1) + (-1) \times (-1) = 0$	Multiplicative identity
$1 + [(-1) + (-1) \times (-1)] = 1 + 0$	Add both sides to 1.
$[1 + (-1)] + (-1) \times (-1) = 1 + 0$	Associative law
$0 + (-1) \times (-1) = 1 + 0$	Definition of -1
$(-1) \times (-1) = 1$	Additive identity

My Sanskrit-Based Demonstration

i.e. $^{-}1$ subtracted 1 time from 0 = $^{+}1$

© 2019 Jonathan J. Crabtree

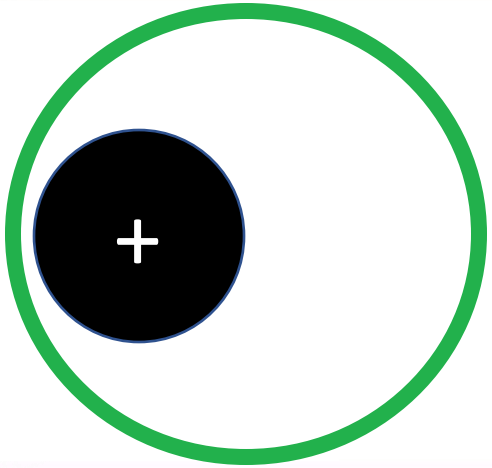
Seeing a Simple Proof for Brahmagupta's ML2

The product of two 'dashed numbers' is positive.

$^{-}a \times -b = {}^{-}a$ *subtracted from zero* b *times*

$^{-}1 \times -1 = {}^{-}1$ *subtracted from zero* 1 *time*

$^{-}1 \times -1 = {}^{+}$



$1 + (-1) = 0$	Definition of -1.
$-1 \times [1 + (-1)] = -1 \times 0$	Both sides multiplied by -1.
$(-1) \times 1 + (-1) \times (-1) = 0$	Distributive law
$(-1) + (-1) \times (-1) = 0$	Multiplicative identity
$1 + [(-1) + (-1) \times (-1)] = 1 + 0$	Add both sides to 1.
$[1 + (-1)] + (-1) \times (-1) = 1 + 0$	Associative law
$0 + (-1) \times (-1) = 1 + 0$	Definition of -1
$(-1) \times (-1) = 1$	Additive identity

My Sanskrit-Based Demonstration
i.e. $^{-}1$ subtracted 1 time from 0 = $^{+}1$

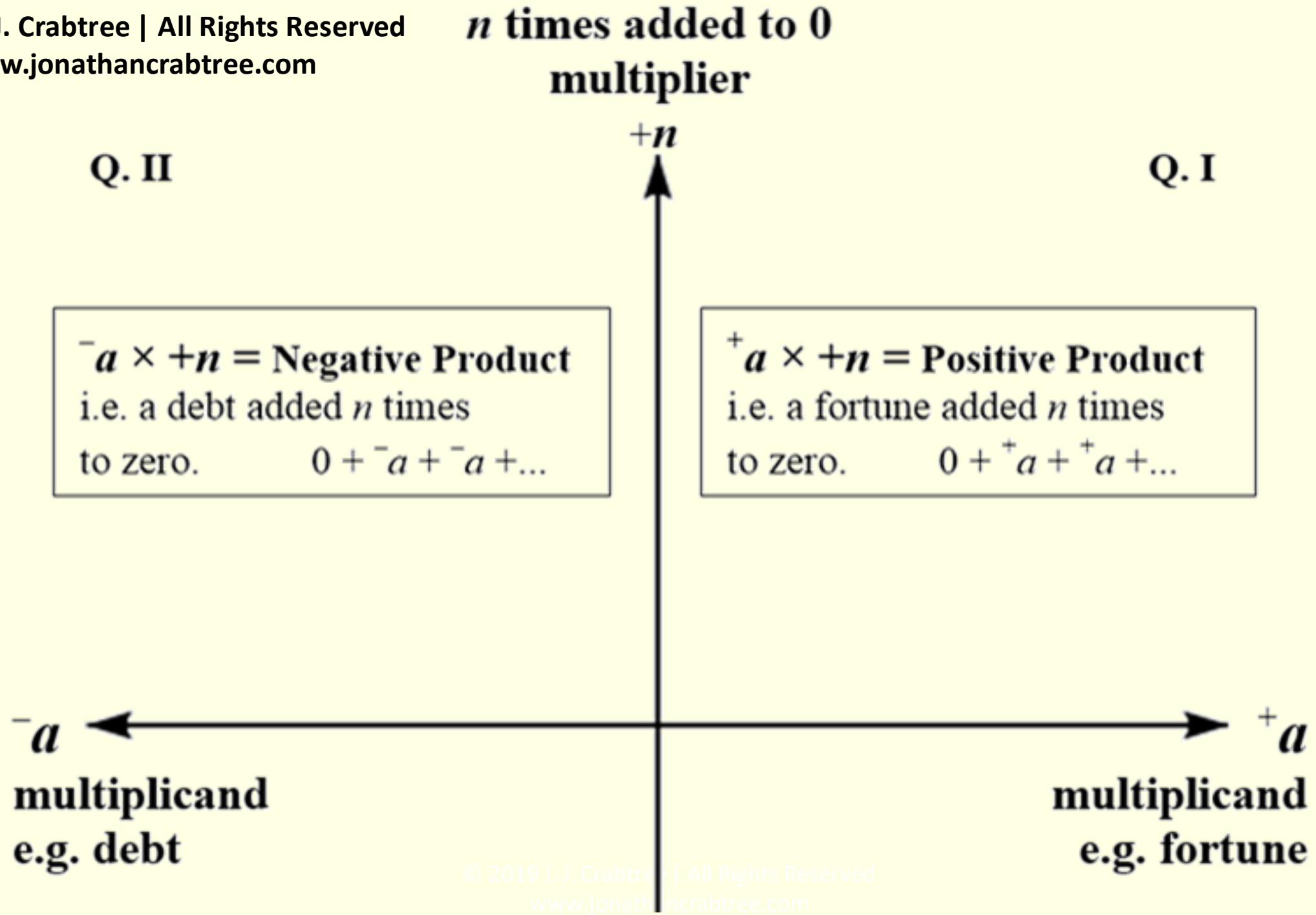
My Western Demonstration

© 2019 Jonathan J. Crabtree

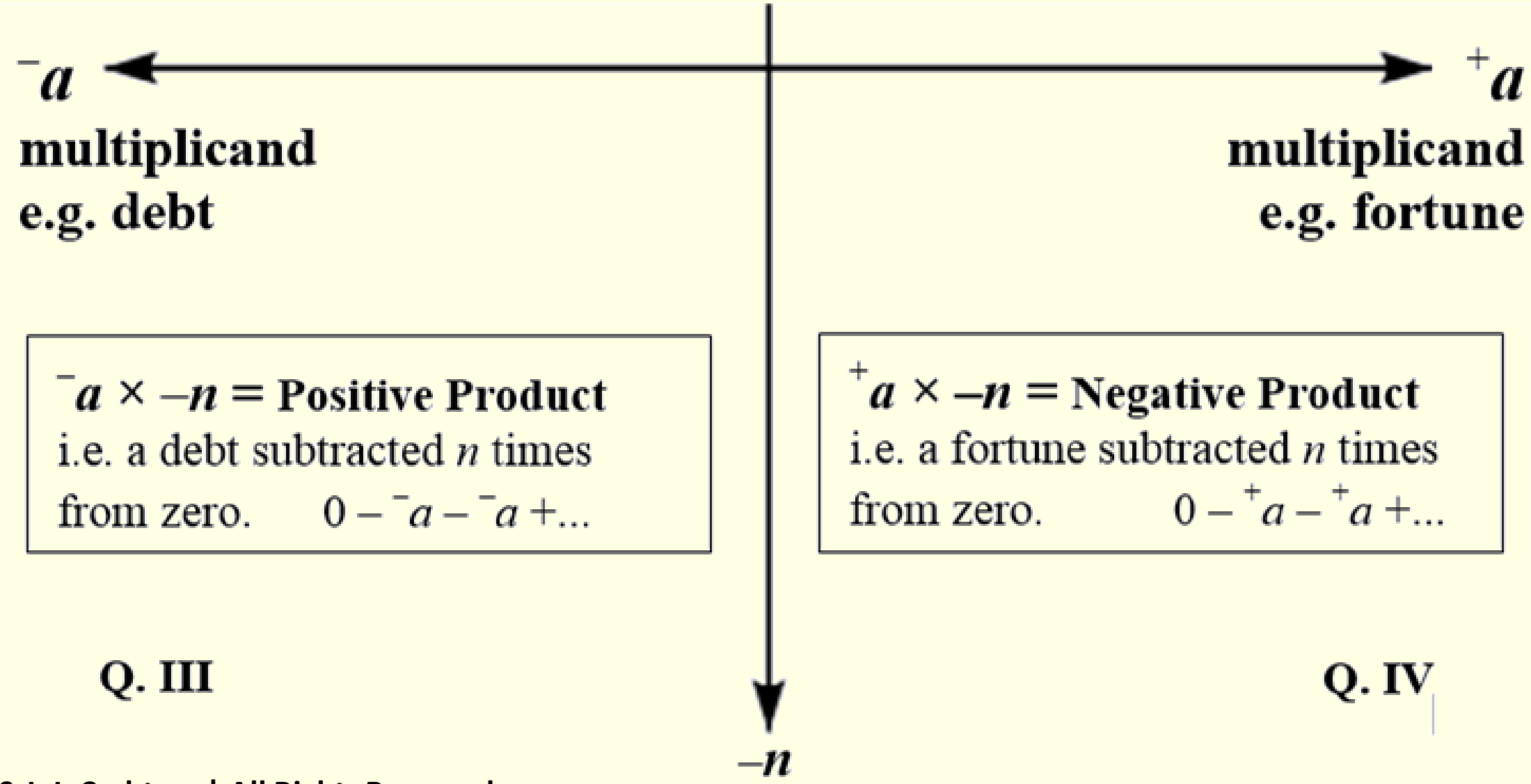
With our corrected understanding of Integer arithmetic, we can now depict it on a Brahmaguptan Plane.

Download the Paper from
www.j.mp/BrahmaguptanPlane

© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

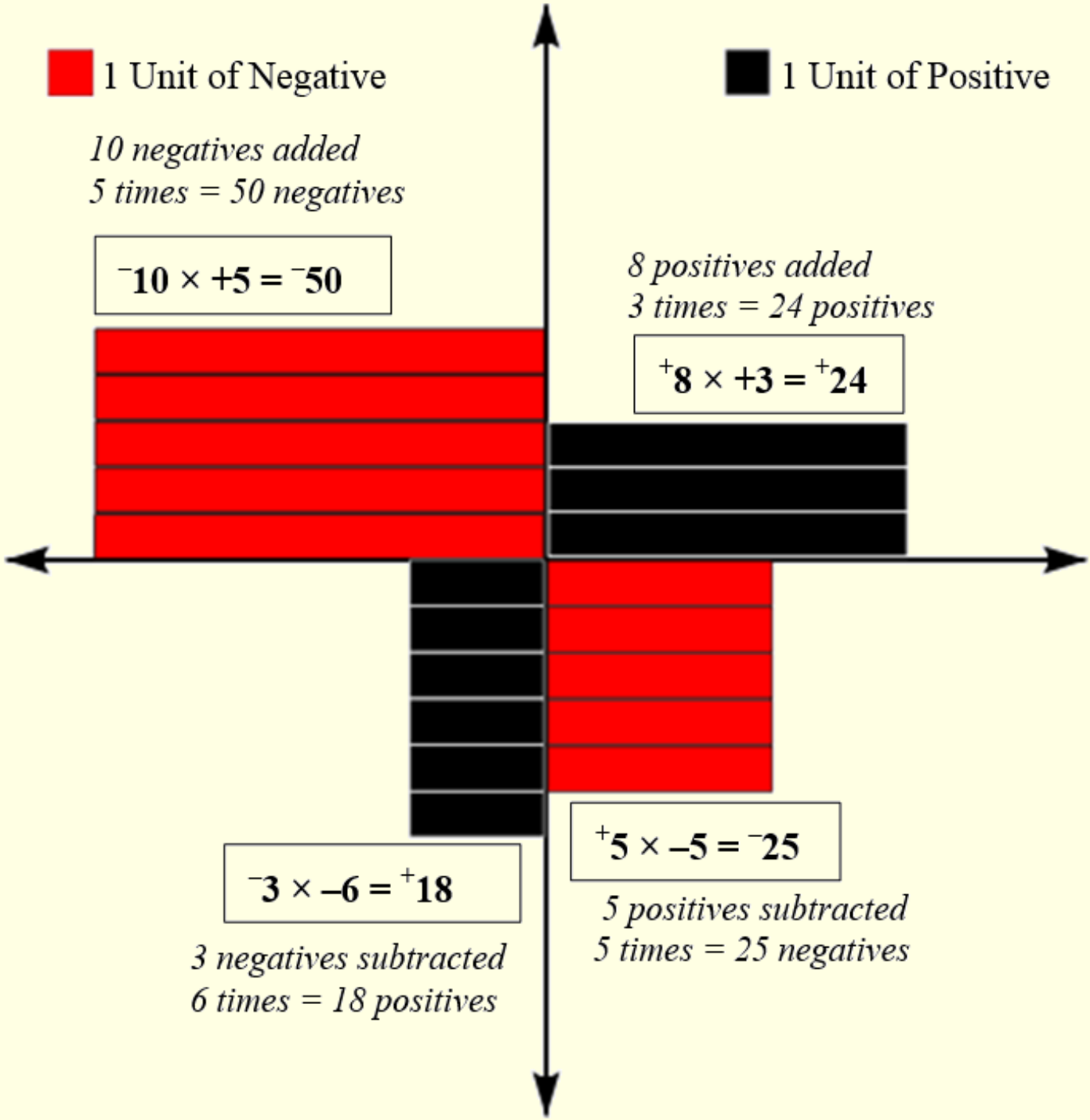


© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com



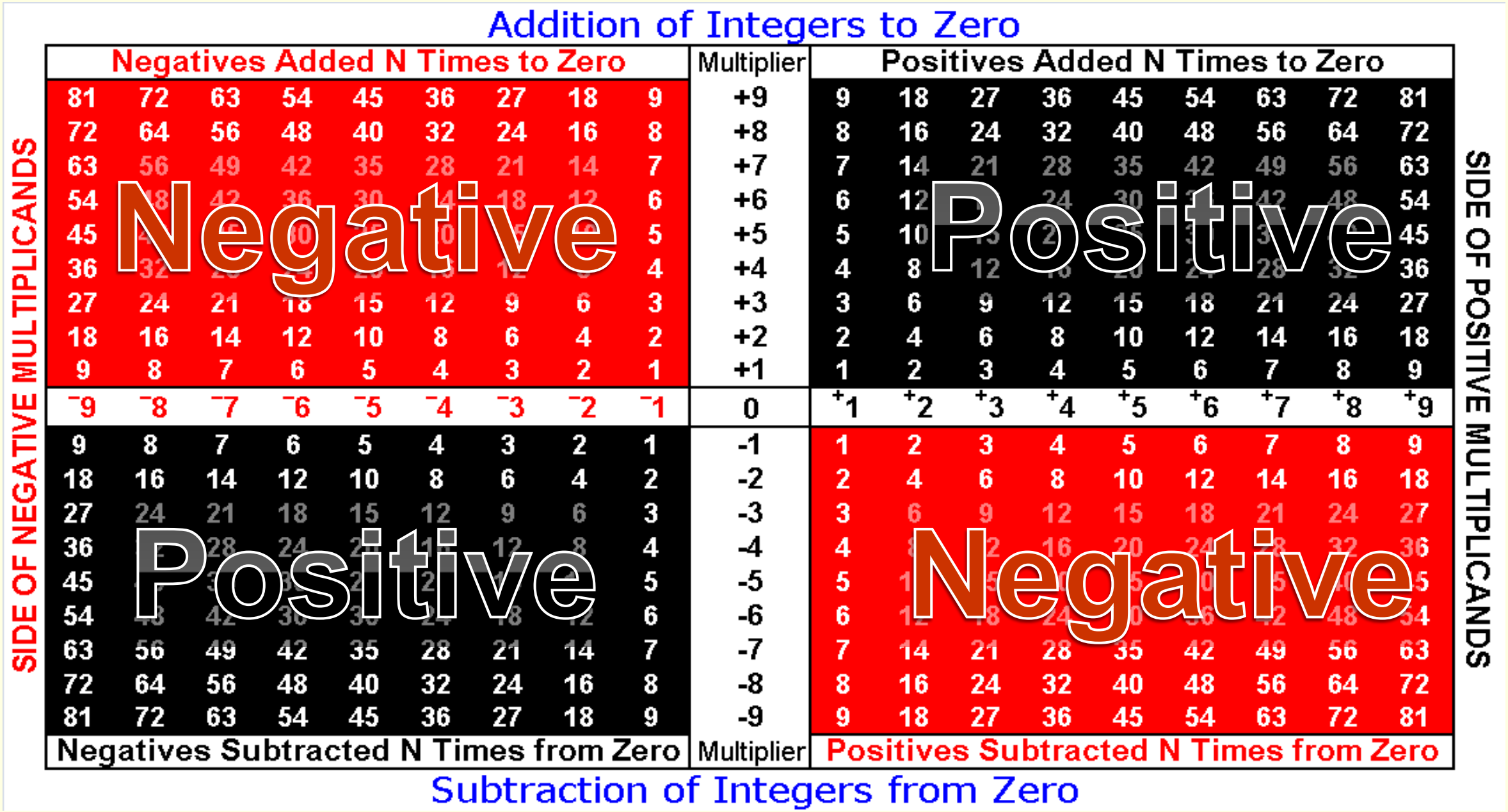
© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

The Brahmaguptan Plane with both positive and negative areas.



© 2019 J. J. Crabtree | All Rights Reserved
www.jonathancrabtree.com

Jonathan J Crabtree's 'Brahmaguptan Plane' with positive and negative areas.

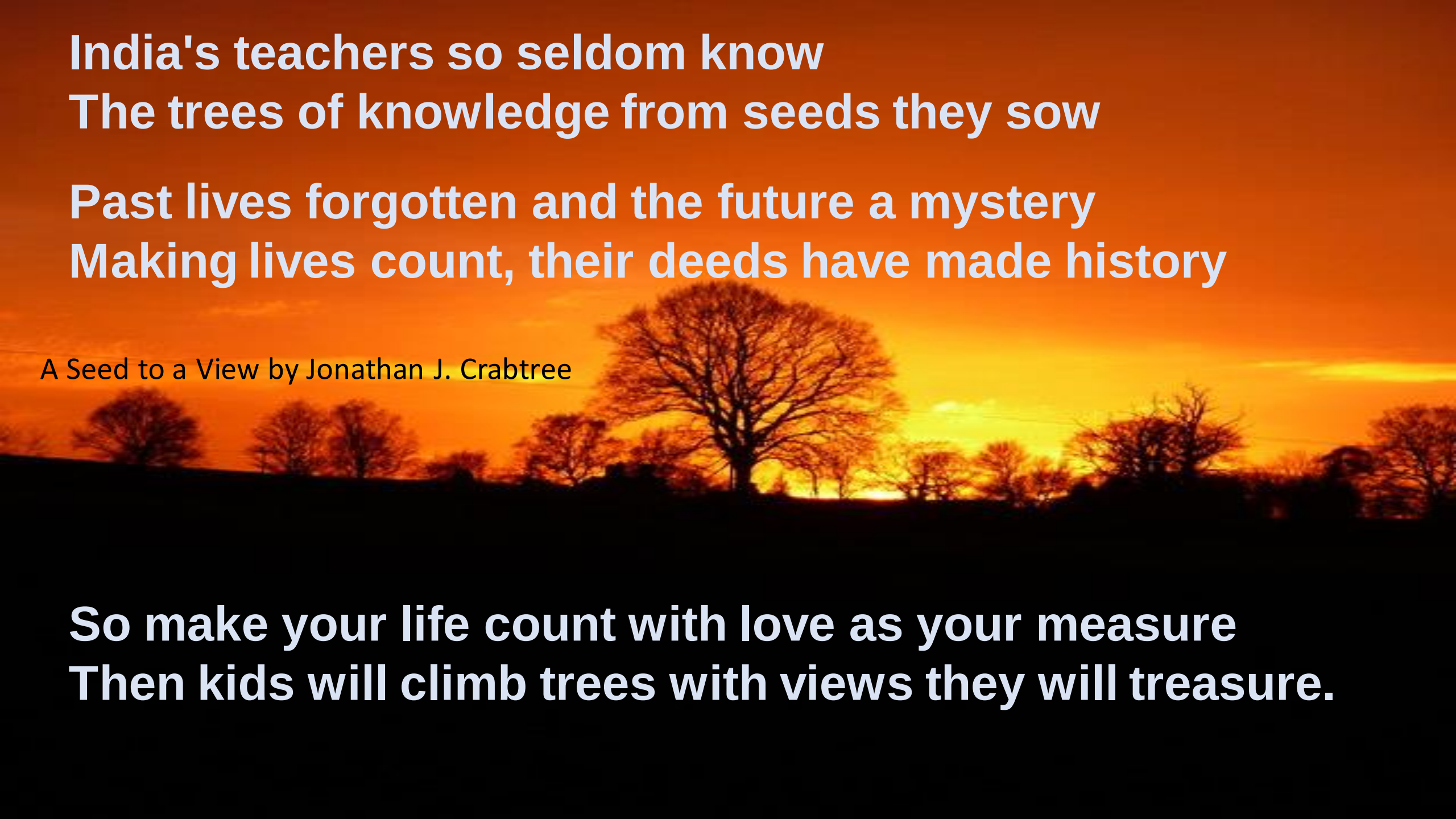


Representations of Negative and Positive Quantities on a 'Brahmaguptan Plane' for India's Primary Classes

Jonathan J Crabtree

www.j.mp/BrahmaguptanPlane

Abstract: *Children's fear of maths is often associated with the introduction of negative numbers. By way of example, asking adult non-mathematicians for the answer to 'negative seven minus negative four' usually results in a wrong answer. However, asking the same question to 12-year-old children in the form What does seven negatives minus four negatives equal? usually results in the right answer. Why is the difference in comprehension so dramatic? In the problematic expression negative seven minus negative four the syntactic structure is adjective adjective verb adjective adjective. With the absence of a noun, the meaning of such maths for most children is lost. Instead, children (and adults) cling to rules memorised without meaning, such as 'two minuses make a plus'. So, what can we do? The answer is simple. We return to 7th Century writings of India, where we discover the astronomer Brahmagupta documented 'adjective-noun' style laws of sign, not for abstract numbers, but for positive quantities, negative quantities and zero. With this insight, we depict simple object-oriented representations of integer arithmetic involving positive and negative quantities. Such a quantitative pedagogy is concrete in nature, yet isomorphic to 'signed numbers.' Therefore, a solid intuitive foundation of integer arithmetic can be laid. Upon this foundation more abstract structures can be built. The integer teaching model that emerges is called the 'Brahmaguptan Plane'.*



India's teachers so seldom know
The trees of knowledge from seeds they sow

Past lives forgotten and the future a mystery
Making lives count, their deeds have made history

A Seed to a View by Jonathan J. Crabtree

So make your life count with love as your measure
Then kids will climb trees with views they will treasure.

Thank you!

If You Care Please Share

www.j.mp/IndiasMaths

Please email me your feedback
feedback@podo.in

Jonathan J. Crabtree
Elementary Mathematics Historian, Melbourne, Australia
info@podometic.in
© 2019 Jonathan J. Crabtree